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This makes it imperative to provide for quick publication of a continually increasing bulk of material, in such a form that it may be easily available to workers in the same field and come to their notice without the necessity of scanning huge piles of periodicals of the most varied description. The system of distribution of 'preprints' among physicists pursuing similar lines of research is an emergency measure which only fulfils this purpose very imperfectly, and clearly demonstrates the need for a more rational organisation of the publication of original papers in various branches of physics.

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L. ROSENFELD

Nuclear Physics
1 (1956)

STUDIES WITH THE SCINTILLATION SPECTROMETER

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Received 8 August 1955

Abstract: The setting-up of a single crystal (Na I(Tl) spectrometer is described. The efficiency of the well-type crystal for gamma-rays has been determined by the summing technique. In the gamma-ray spectrum of I^{128}, in addition to the wellknown 440 keV gamma-ray, a γ-ray of energy 960 ± 6 keV and two feeble γ-rays of energy 520 keV and 750 keV have been found. It has been argued that the 750 keV/γ-ray arises in the electron-capture branching, and the 960 keV and 520 keV/γ-rays arise in the β^--branching in the decay of I^{128}.

1. Introduction

During the last few years, the scintillation spectrometer has proved extremely powerful in the detection and measurements of γ-radiation. For the drawing up of decay schemes of radioactive isotopes, one often needs to perform coincidence experiments to find the radiations in cascade. For this, one needs more than one spectrometer unit. But, by employing a large enough thallium-activated sodium-iodide crystal with a hole at the top, a single crystal spectrometer can perform many functions of a coincidence setup. Here the setting up of a scintillation spectrometer with an ordinary and a well-type crystal, the estimate of the photoelectric efficiency in the crystal for γ-rays of different energies by the summing technique, and the studies of the γ-rays emitted in the decay of 25 min. I^{128} are described.

The scintillation spectrometer consists of a Na I (Tl) crystal mounted on an E.M.I. photo-multiplier type 6260, a preamplifier, a modified non-overload Bell and Jordan amplifier, a single channel analyser designed by Fairstein and Porter [1]) with a minimum window width of 0.75 volts, a scalar, a highly stabilized negative high voltage supply. The differentiating time of the amplifiers was 2μ sec. Because of the saturation effect in the E.M.I. photo-multipliers, the last three dynodes were connected to serve as one dynode, and 125 to 150 volts are applied. The crystals employed were

"Harshaw Chemical" hermetically-sealed crystal $1\frac{1}{2}$" in diameter and $\frac{1}{2}$" thick, and a "Larco" well-type crystal $1\frac{3}{4}$" in diameter 2" thick with a hole at the top of diameter 5/8" and depth $1\frac{1}{2}$". The spectrometer was calibrated with the radiations from Zn^{65}, Co^{60}, and Cs^{134}.

2. The Summing Technique

This technique has been used recently by Lu and Wiedenbeck [2]) and Lu, Kelly and Wiedenbeck [3]) to find the γ-rays in cascade, and in the case of Ba^{131} to find the conversion coeffcient of

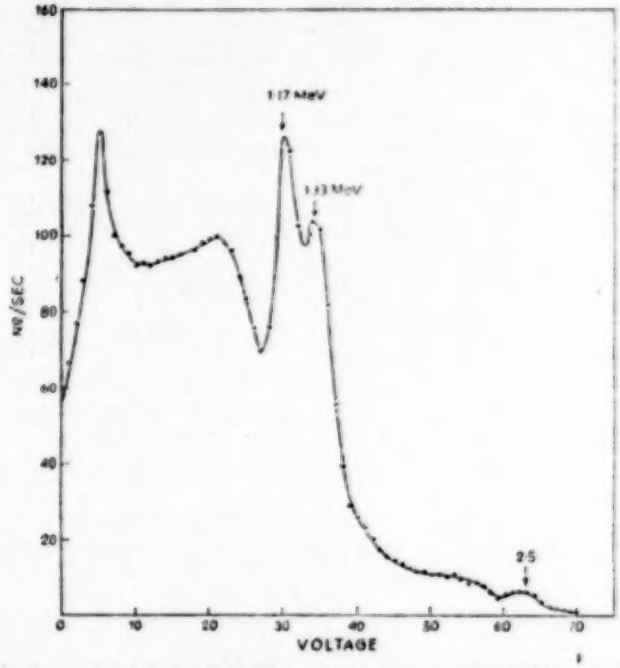

Fig. 1. Scintillation spectrum of Co^{60} with the source inside the well showing the photo-peaks of the cascade γ-rays, the sum-peak and the hump in between.

122 keV γ-ray. Although the Larco-make well-type crystal available to us is not of really adequate size for proper application of the summing technique, yet the method has been applied here to find the overall photo-electric efficiency of the crystal for the cascade γ-rays of various energies. When one is studying, by placing inside the well of a large Na I (T1) crystal, a radio-active source, which emits two γ-rays, γ_1 and γ_2 in cascade, of energies E_1 and E_2, one gets, in addition to the two photo-peaks, a sum peak and a broad hump in between, as can be seen from the scintillation spectrogram of Co^{60} and Cs^{134} radiations shown in fig. 1 and fig. 2.

The photo-peaks of the γ-rays γ_1 and γ_2 arise due to the photo-electric absorption and due to the simultaneous absorption of the Compton electron and the scattered quantum. These two processes together constitute the effective photoelectric absorption. Inside the well of the crystal, the two gamma-rays may be absorbed by the effective photo-electric process within the pulse duration time of the scintillation spectrometer, which is about a μ sec. and a peak at the energy $(E_1 + E_2)$ appears. It may happen inside the well of the crystal, that γ_1 may be absorbed by the effective photo-

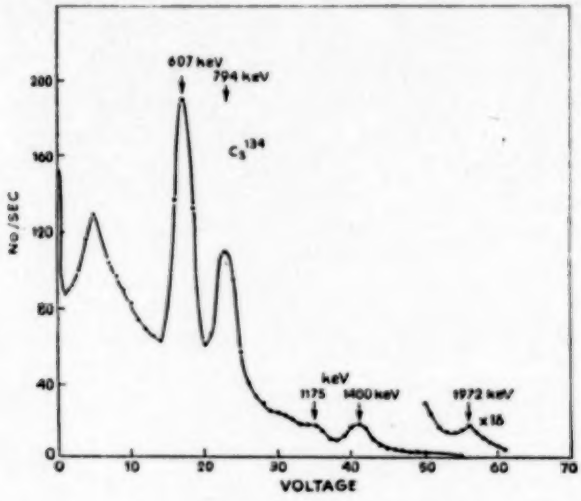

Fig. 2. Scintillation spectrum of Cs^{134} with the source inside the well of the crystal, showing the photo-peaks of 607 and 794 keV γ-rays, the sum-peak at 1.4 MeV, and the hump in between.

electric process simultaneously with the Compton electron of γ_2 and vice versa. This will give rise to pulses of heights larger than E but smaller than $(E+E_c)$ where E_c is the maximum Compton electron energy of the other γ-ray. Hence these pulses will cause a broad hump in between the photo-peaks and the sum peak. The simultaneous detection of the Compton electron of the two cascade γ-rays gives rise to a continuous distribution of pulses ranging from very small pulses to the maximum at $(E_{c1} + E_{c2})$; a small portion of these will fall in the region of the broad hump.

Let the area under the photo-peaks of the γ-rays γ_1 and γ_2 be A_1 and A_2 respectively, and the area of the spectrum formed by the summing up of the photo-electron of one γ-ray and the Compton electrons of the other be C_1 and C_2, and the area under the sum peak be S. Now, $N\epsilon$ would be the area under the photo-peak, if

no summing-up takes place. When the source is inside the well of the crystal,

$$N\epsilon_1 \sim A_1 + C_1 + S$$
$$N\epsilon_2 \sim A_2 + C_2 + S$$
$$N\epsilon_1\epsilon_2 \sim S.$$

If one knows the form of the Compton spectrum of the γ-rays γ_1 and γ_2, one can break up the broad hump between the photopeaks and the sum peak, and so estimate C_1 and C_2. If these areas are corrected for the constant window width of the single channel analyser, one can calculate the effective photoelectric efficiencies of the γ-rays by the relation

$$\epsilon_1 = \left(\frac{E_1}{E_1 + E_2}\right)^{1/2} \frac{S}{A_2}\left(1 - \frac{C_2 + S}{A_2}\right)$$

deduced from the above three relations. Here C_1 and C_2 and $S \ll A_1$ and A_2. The factor $\left(\frac{E_1}{E_1 + E_2}\right)^{1/2}$ is the correction due to the constant window width of the single channel analyser [3]). From the sum spectra reproduced in fig. 1 and fig. 2, the effective photoelectric efficiencies have been estimated of 1.33 MeV and 1.17 MeV γ-rays of Co^{60} and of 796 keV and 605 keV γ-rays of Cs^{134} in the well-type crystal to be 1.5 %, 2 %, 5.9 % and 9.7 % respectively.

3. The Study of 25 min I^{128}

It has been found by Siegbahn and Hole [4]) that I^{128} decays by β^--emission to the ground-state of xenon with the maximum β^--energy of 2.02 MeV. In about 7 % cases, β^--emission takes place to an excited state of xenon followed by a γ-ray of energy 428 keV, measured by them in a lens spectrometer using a lead radiator. Wapstra, Verster and Boelhouer [5]) studied Cs^{128} with a scintillation spectrometer and found a γ-ray of energy nearer 500 keV than 428 keV. Therefore they studied I^{128} with a scintillation spectrometer and found the γ-ray energy to be 455 $\pm$ 5 keV. They found another γ-ray associated with I^{128} of energy 980 $\pm$ 30 keV. In the decay of Cs^{128}, they found a γ-ray of energy about 1.5 MeV. Germagnoly *et al.* [6]) studied I^{128} with a scintillation spectrometer, and found the γ-ray energy to be 436 keV; but they do not report the γ-ray at 980 keV. Barber [7]) studied I^{128} and found less than $1\beta^+$ per 1000 β^-'s. Reynolds [8]) showed by a mass spectrometric analysis of an old neutron-irradiated sample of

iodine that I^{128} has about 5 % electron-capture and positon emission branching. Mims and Halban [9]) studied x-rays emitted by I^{128} and estimated the percentage of K-capture to be 6.5 ± 0.7.

The source of I^{128} was prepared by the Szilard Chalmers process with neutrons from the Ra-Be capsule and the Philips neutron generator. Sources of strength upto 100 μC strength were prepared in this way. The spectrum of the γ-rays from I^{128} was taken with the scintillation spectrometer with the I^{128} source both inside and outside the well of the crystal. In fig. 3 is shown the spectrum with the source inside.

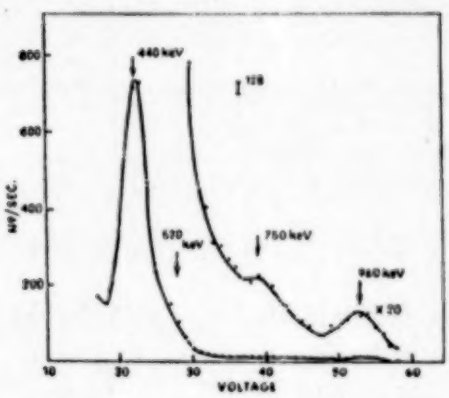

Fig. 3. The scintillation spectrum of I^{128}, showing the photo-peaks at 440 and 960 keV, and perhaps also at 520 and 750 keV.

There are two prominent peaks, one at 440 ± 5 keV, and the other at 960 ± 15 keV. In addition to these, there are perhaps very weak intensity radiations of energy round 520 keV and 750 keV. For the exact determination of the energy of these radiations, 0.411 MeV line of Au^{198}, 0.794 MeV line of Cs^{134}, and 1.117 MeV line of Zn^{65} were used. The apparent intensity of the 960 keV line in comparison with the 440 keV line is 8 per 1000. The ratio of the photoelectric efficiency of the crystal for the 440 keV γ-ray to the 960 keV γ-ray has been estimated to be 4. The true intensity of the 960 keV γ-ray then comes out to be 3 % of the 440 keV γ-ray, and two per thousand disintegrations.

As to the origin of the 960 keV radiation, it could arise from some electron-capture in I^{128} leading to the first excited state of Te^{128}, or due to some β^--transition leading to the second excited state of Xe^{128}. From the plot of the decay energy of the even mass number isotopes ${}_{52}Te \rightleftarrows {}_{53}I$ against the neutron number in tellurium isotopes [10]), one finds (the decay energies are taken from King [11])) that the decay energy of I^{128} to Te^{128} is about 1.4 MeV. The energies of the first excited states of ${}_{50}Sn$ even isotopes, and of the first and the second excited states of ${}_{52}Te$, ${}_{54}Xe$, and ${}_{56}Ba$ even isotopes have been plotted against their neutron number [12] in fig. 4.

From the curves in the above figure, one can see that the first excited state of Te^{128} is at about 800 keV. It is difficult to push

the extrapolated point to 960 keV without making the curve unsymmetrical with respect to the curve for xenon, which shoots up only very near $N = 82$. There is no apparent reason for the first excited state of Te^{128} being higher than 800 keV. It seems, therefore, that the 960 keV γ-ray is not associated with the electron-capture branching in I^{128}. On the other hand, the second excited

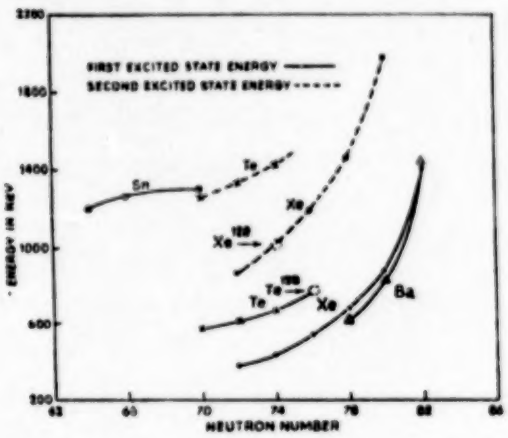

Fig. 4. The first excited state energies (full line curves) and the second excited state energies (dotted line curves) plotted against the neutron number. The first excited state energy of Te^{128} at about 750 keV is shown with a dotted circle on the curve for tellurium first excited state, and the second excited state energy of Xe^{128} is shown with a dotted circle on the curve for the second excited states for xenon.

state of Xe^{128} can be seen in the above figure to lie at about 950 keV. One is then led to think that this γ-ray is due to some β^--transition to the second excited state of xenon, followed by a γ-transition to the ground state. The existence of a γ-ray of energy about 520 keV is suspected; and it may very well be that this feeble radiation, with an intensity not more than 1.5 % of the 440 keV radiation, may be due to the transition from the second excited state of Xe^{128} to the first one. Since the 520 keV and the 440 keV γ-rays would, on this basis, be in cascade, a considerable summing effect should be observed when the source is placed inside the well of the crystal; the relative intensity of the 960 keV peak, when the source is inside the crystal, should be somewhat larger than when the source is outside. The upper limit of the contribution of summing to the 960 keV peak has been estimated to be about 10 %. The γ-ray of energy about 750 keV whose presence is also suspected with an intensity of about 0.5 % of the intensity of the 440 keV γ-ray, could very well arise from some

electron-capture in I^{128} leading to the first excited state of Te^{128} which is predicted in fig. 4 to lie at about 800 keV. The decay scheme of I^{128} based on the above considerations is given in fig. 5. On this basis, the log *ft* values for β^--emission to the ground state, the first excited state and the second excited state of Xe^{128} calculated with Moszkowski's nomogram [13]), come out to be 5.9, 6.5 and 7.8 respectively; and the log *ft* values for the electron-capture to the ground and the first excited states of Te^{128} are found to be 5.5 and 6.5 respectively.

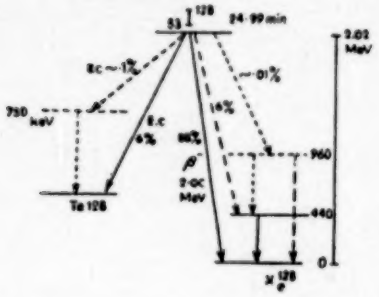

Fig. 5. The probable decay scheme of I^{128}. One can compare this with the decay scheme of I^{126} (Wu *et. al.* Phys. Rev. **98** (1955) 1230).

Our grateful thanks are due to Dr. H. J. Bhabha, F.R.S., for his interest. We express our thanks to Dr. R. Ramanna and his group for carrying out the neutron irradiation. We thank Dr. A. Mukerji and Dr. B. V. Thosar for helpful discussions. We are very grateful to Mr. B. K. Madan for his ungrudging help.

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Note added in the proof

In the Bulletin of the American Physical Society, Mexico meeting [Aug. 29-31, 1955] Wu *et al.* have given the results of their investigations on I^{128}. Their results agree n essence with our findings.

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1 (1956)

INTERPRETATION OF (d,t) ANGULAR DISTRIBUTIONS

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Abstract: Expressions derived from stripping theory by Butler and Salpeter are used to obtain the upper limit of 0.11 for the probability of finding the triton in the configuration of deuteron ground state plus second neutron, utilizing the experimental data on angular distributions for the protons in C^{12} (d,p) C^{13} (J. Rotblat [5])) and the tritons in C^{13} (d,t) C^{12} (Holmgren *et al.* [4]))

1. Introduction

It is now well established [1, 2]) that the angular distributions of (d,t) and (t,d) nuclear reactions contain large contributions from pick up or stripping respectively at small angles. In ref. [1]) it was further pointed out that, if one assumes the cross-sections for such a reaction to be given by the stripping formula [3]), and one also has available experimental cross-sections for both a (d,t) and a (p,d) reaction, for example, between the same two nuclear levels, then it is, in principle, possible to obtain some quite detailed information concerning the structure of the triton.

More precisely, one can obtain:
(a) the probability, written as $|A_0|^2$, of the triton having the configuration of deuteron ground state plus second neutron and (b) the momentum distribution of the second neutron in the above configuration.

The purpose of this work is to follow up the above, making use of the recent experimental results of Holmgren *et al.* [4]) for the reaction C^{13} (d,t) C^{12}, and also that of Rotblat [5]) for the reaction C^{12} (d,p) C^{13}.

No information, other than that which can be readily deduced, has been obtained on point (b). The reason is that, for the

† Also supported by the Nuclear Research Foundation within the University of Sydney.

C^{13} (d,t) C^{12} reaction at the energies employed by Holmgren *et al.* (deuteron laboratory energy of 3.29 MeV) the theoretical angular distribution requires a knowledge of the momentum distribution for the second neutron only for quite small momenta, which is immediately obtainable from the difference in binding energy of the triton and deuteron.

The main result obtained from the experimental data is the value of the product $|A_0|^2 C_T^2$, where C_T is a normalization factor of the wave function of the second neutron, when the other two particles form a deuteron in its ground state. An estimated lower limit for C_T^2 then yields the quite small upper limit of 0.11 for $|A_0|^2$, i.e. for the probability of finding the triton in the configuration deuteron ground state plus second neutron.

This allows one to draw the conclusion that any theoretical attempt to discuss the triton — and, indeed, also (n,d) and (p,d) scattering — by methods which rely on having an unpolarised deuteron plus second neutron, are completely unjustified.

In connection with the three-body scattering problem, this conclusion is quite far reaching, in that all theoretical methods developed for calculating these cross-sections have in fact ignored the deuteron polarization [6a, 6b, 6c, 7a, 7b]).

It is, of course, well known that Coulomb and nuclear corrections can modify the cross-section given by simple stripping theory [3]), particularly with respect to the absolute magnitude [8, 9]), and that this could change the value obtained for $C_T^2 |A_0|^2$ somewhat. Because of this, the precaution of obtaining $C_T^2 |A_0|^2$ at a variety of angles (by comparison of theoretical and experimental cross-sections) has been taken, and it is indeed found to be very closely constant over all angles involved in the forward structure of the angular distribution (i.e. those parts of the angular distributions which come from stripping or pick-up).

The angular dependence of both the experimental cross-sections must therefore be given extremely well by the stripping theory formulae. The possibility that the absolute magnitude of the cross-section might be changed by the above corrections cannot be ruled out, but here one knows [8, 9]) that for light elements such as C^{12}, C^{13}, and for the sort of energies involved in the reactions here considered, the maximum reduction to be expected in the cross-section is about a factor of 2. Since, however, only the ratio of the two cross-sections is concerned, such a possible uncertainty would

very largely be eliminated, and cannot affect the main conclusion that $|A_0|^2$ must be small.

2. Stripping and Pick-up Cross Sections

The differential cross-section for a (d,p) stripping reaction, in which there is one allowed value l_n for the angular momentum of the captured neutron is [3, 10])

$$\sigma(\mathrm{d,p}) = \frac{m_d^* m_p^* K_p}{2\pi^2 m_n^{*2} K_d q} \frac{(2J+1)}{(2j+1)} \Phi_d(|\tfrac{1}{2}\mathbf{K}_d - \mathbf{K}_p|)^2 \frac{|f(J, j, l_n; r_0)|^2}{(h_{l_n}^{(1)}(iKr_0))^2}$$
$$\times [W\{r_n j_{l_n}(|\mathbf{K}_d - M_1/M_2\,\mathbf{K}_d|),\ h_{l_n}^{(1)}(iKr_n)\}_{r_n=r_0}]^2. \qquad (1)$$

The notation here is the following:

m_d^*, m_p^*, m_n^* — appropriate reduced masses of the incident deuteron, outgoing proton and captured neutron respectively;

K_d, K_p — wave numbers for the incident deuteron and outgoing proton, in the centre of mass system, respectively;

K — wave number corresponding to the binding energy of the captured neutron;

M_1, M_2 — masses of the initial and final nuclei respectively.

J, j — spins of the final and initial nuclei respectively;

r_0 — radius of initial nucleus;

W — the Wronskian, derivatives being taken with respect to r_n at $r_n = r_0$;

j_{l_n} — spherical Bessel function;

$h_{l_n}^{(1)}(iKr_n)$ — spherical Hankel function, defined by

$$h_{l_n}^{(1)}(iKr_0) = e^{-K(r_n - r_0)} \sum_{n=0}^{l_n} \frac{(l_n + n)!}{n!(l_n - n)!} (2Kr_n)^{-n};$$

Φ_d — Fourier transform of the internal deuteron wave function, normalized to unity;

$|f(J, j, l_n; r_0)|^2$ — reduced width, [10]) viz. the probability density for finding a neutron with orbital angular momentum l_n at the surface of the final nucleus, when the initial nucleus is left in its ground state;

q — a number slightly greater than unity — see ref. [10]).

The differential cross-section for a (t,d) stripping reaction with one particular value l_n for the angular momentum of the captured neutron can be found in exactly the same way as for eq. (1), with the result

$$\sigma(\mathrm{t,d}) = \frac{3m_T^* m_d^* K_d}{4\pi^2 m_n^{*2} K_T q} \frac{|A_0|^2}{2j+1} \frac{2J+1}{2j+1} \{\Phi_T(|{}^2/_3\mathbf{K}_T - \mathbf{K}_d|)\}^2$$

$$\times \frac{|f(J, j, l_n; r_0)|^2}{\{h_{l_n}^{(1)}(iKr_0)\}^2} [W\{r_n j_{l_n}(|\mathbf{K}_T - M_1/M_2\mathbf{K}_d|), h_{l_n}^{(1)}(iKr_n)\}_{r_n=r_0}]^2. \quad (2)$$

The notation in (2) is the same as, or an obvious extension of, that of eq. (1), except for the form factor $(\Phi_T)^2$, and the multiplying constant $|A_0|^2$. These may be described as follows.

Expand the (unsymmetrized) triton wave-function in terms of states t of the deuteron, thus:

$$\psi_T(\mathbf{r}, \mathbf{R}) = \sum_{ti} A_{ti}\chi_{ti}(\mathbf{r})f_{ti}(\mathbf{R}). \quad (3)$$

Here $\mathbf{r}$ is the difference between the coordinates of the proton and a neutron, and $\mathbf{R}$ is the position vector of the second neutron with respect to the midpoint of $\mathbf{r}$. The $\chi_{ti}(\mathbf{r})$ are wave-functions of states t of the deuteron with spin orientation i, and both χ_{ti} and f_{ti} are assumed to include spin functions and to be normalized to unity — as also is ψ_T. Then the function Φ_T is merely the Fourier transform of $f_{0i}(\mathbf{R})$ ($t = 0$ corresponding to the deuteron ground state), which is independent of i. Also, the constant $|A_0|^2$ is defined to be $\Sigma_i |A_{0i}|^2$; and is thus the total probability of finding the triton in the configuration of a deuteron and third particle.

From eq. (1) and (2) we see that the ratio of the cross-sections for reactions between the same two energy levels is independent of the reduced width $|f|^2$, and, apart from the angular momentum l_n and radius r_0, is dependent only on specific properties of the deuteron and triton, viz. the internal momentum distribution of the deuteron Φ_d, the probability factor $|A_0|^2$, and the momentum distribution Φ_T.

The ratio of the stripping cross-sections for a (d,t) and a (d,p) reaction (the particular ratio considered in the next section) between the same nuclear levels is therefore, using detailed balance on eq. (2),

$$\frac{\sigma(\mathrm{d,t})}{\sigma(\mathrm{d,p})} = \frac{2j+1}{2J+1}\,|A_0|^2\,\frac{m_T^*}{m_p^*}\,\frac{(m_d^*)_{\mathrm{I}}}{(m_d^*)_{\mathrm{II}}}\,\frac{(K_T/K_d)_{\mathrm{I}}}{(K_d/K_p)_{\mathrm{II}}}\,\frac{(\Phi_T^2)_{\mathrm{I}}}{(\Phi_d^2)_{\mathrm{II}}} \times \frac{[W\{r_n j_{l_n\mathrm{I}},\ h_{l_n}^{(1)}(iKr_n)\}_{r_n=r_0}]^2}{[W\{r_n j_{l_n\mathrm{II}},\ h_{l_n}^{(1)}(iKr_n)\}_{r_p=r_0}]^2}, \tag{4}$$

where the subscripts I and II signify that momenta, etc., appropriate to the (d,t) and (d,p) reaction respectively are used.

It is clear that if the experimental ratio of the cross-sections is known, and if Φ_T is found from a (d,t) angular distribution †), then one can find $|A_0|^2$ directly since the quantities Φ_D and C_D are known.

In the case of the (d,t) experiment with which we compare in the next section, however, the energies are such that one can obtain information on $\Phi_T(k)$ for small values of k only, and it is found that this behaviour agrees with what one would expect from the known asymptotic form of the function $f_0(\mathbf{R})$ (see eq. (6) below). Thus one does not have enough information from experiment to determine the normalization constant $C_T/2\sqrt{\pi}$, which normalizes the function $f_0(\mathbf{R})$ to unity (with respect to integration over $\mathbf{R}$).

Thus all one is really able to determine from a ratio of cross-sections is the quantity $C_T^2\,|A_0|^2$, with the normalization factor C_T unknown.

However, a lower limit to C_T^2, and thus an upper limit to $|A_0|^2$, can be found by assuming the unpolarized deuteron plus second neutron (resonating group) model of the triton. For this purpose one assumes the following form for ψ_T:

$$\psi_T \approx \chi_0(\mathbf{r})f_0(\mathbf{R}).$$

Then f_0 satisfies the equation

$$\nabla_R^2 f_0(\mathbf{R}) = f_0(\mathbf{R})\left\{\lambda^2 + \frac{4M}{3\hbar^2}\int |\chi_0(\mathbf{r})|^2\{V(\mathbf{R}+\mathbf{r}/2)+V(\mathbf{R}-\mathbf{r}/2)\}d\mathbf{r}\right\}, \tag{5}$$

where $\dfrac{3\hbar^2}{4M}\,\lambda^2 = 6.2$ MeV (the difference in triton and deuteron binding energies) and $V(\varrho)$ is the nucleon-nucleon interaction.

† The distribution Φ_T so found would, of course, not be expected to be accurate for very large values of k, corresponding to a small separation between the second neutron and the centre of mass of the other two particles — say $\gtrsim 2 \times 10^{-13}$ cm — since this condition violates the basic assumption of stripping theory.

The asymptotic form of eq. (5) is, of course, simply

$$f_0(\mathbf{R}) \simeq \frac{e^{-\lambda R}}{R}. \tag{6}$$

In order to obtain information on the behaviour of $f_0(\mathbf{R})$ for small R, eq. (5) has been solved numerically for the case

$$V(\varrho) = V_0 e^{-\varrho^2/a^2}$$

and, as usual,

$$\chi_0(r) = C_d(e^{-\alpha r} - e^{-\beta r}), \tag{7}$$

where $|V_0| = 86.4$ MeV and $a = 1.332 \times 10^{-13}$ cm from ref. [7a]) and where α and β in the Hulthen wave function are given by the requirement of best fit to the deuteron binding energy and low energy scattering data.

If one writes

$$f_0(R) = \frac{\mu_0(R)}{R}$$

the result is that $|\mu_0(R)|^2$ has the form shown in fig. 1. Using this function, the normalization constant C_T is given by

$$C_T^2 = \frac{1}{\int_0^\infty \frac{|\mu_0(R)|^2 R^2 dR}{R^2}}. \tag{8}$$

It is clear that the most important feature of $\mu_0(R)$ for the purpose of finding C_T (apart from its asymptotic form) is the position of its maximum. From purely physical arguments, one would expect this maximum to occur at that point where the second neutron (as it is brought up from infinity) begins to interact with the other two particles, i.e. on the resonating group approximation at about the 'radius' of the deuteron.

The position of the maximum of $\mu_0(R)$, as can be seen from fig. 1, is consistent with this rough picture, and indeed from the above physical arguments one would expect to obtain a roughly similar position for the maximum independent of the particular choice of a reasonable function V occurring in eq. (5).

It is also clear, however, that the position of the maximum of $\mu_0(R)$ found in the above way must be an underestimate. When one neutron is 'far' away, the remaining neutron and proton do not always form a ground state deuteron (and one sees in the next section that they rarely do) but are in states of very much larger

spatial extent than the ground state. Thus the wave function of the second neutron will lose its asymptotic form, as R is decreased,

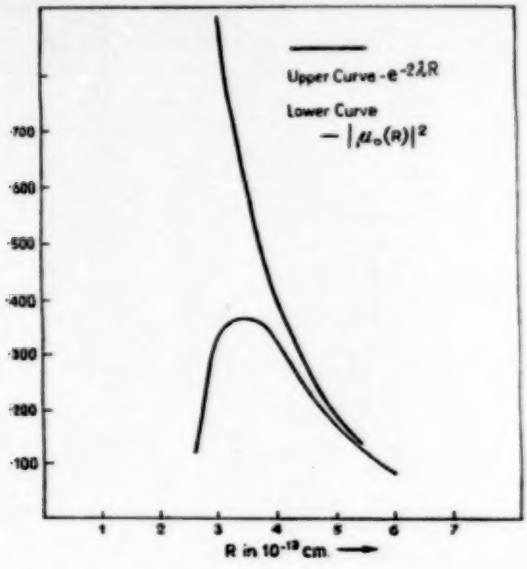

Fig. 1

at much greater values of R than the resonating group approximation would give, and have the maximum of its wave function at a larger distance. Formally, this is seen from the fact that the correct equation for $f_0(R)$, which should replace eq. (5), is

$$\nabla_R^2 f_0(\mathbf{R}) = f_0(\mathbf{R}) \left\{ \lambda^2 + \frac{4M}{3\hbar^2} \int |\chi_0(\mathbf{r})|^2 \{V(\mathbf{R}+\mathbf{r}/2) + V(\mathbf{R}-\mathbf{r}/2)\} d\mathbf{r} \right\}$$

$$+ \frac{4M}{3\hbar^2} \sum_{t \geq 1} A_t/A_0 f_t(\mathbf{R}) \int \chi_0^*(\mathbf{r}) \chi_t(\mathbf{r}) \{V(\mathbf{R}+\mathbf{r}/2) + V(\mathbf{R}-\mathbf{r}/2)\} d\mathbf{r}. \quad (9)$$

The term in eq. (9) additional to eq. (5) requires much larger values of R before it can be ignored, due to the greater extent of the function $\chi_t(r)$, thereby bearing out the above remarks.

In the next section it will, however, be found very useful to employ eq. (8) in giving a lower limit on C_T^2, so enabling an upper limit on $|A_0|^2$ to be given.

3. Comparison with Experiment

We now make use of the recent results obtained by Holmgren *et al.* [4]) for the C^{13} (d,t) C^{12} reaction, and of Rotblat [5]) for the C^{12} (d,p) C^{13} reaction, to obtain the value of the quantity $C_T^2 |A_0|^2$

from experiment (and hence an upper limit to $|A_0|^2$ by using eq. (8).

The experimental results for the differential cross-section of the $C^{13}(d,t)C^{12}$ reaction obtained by Holmgren *et al.* is shown in

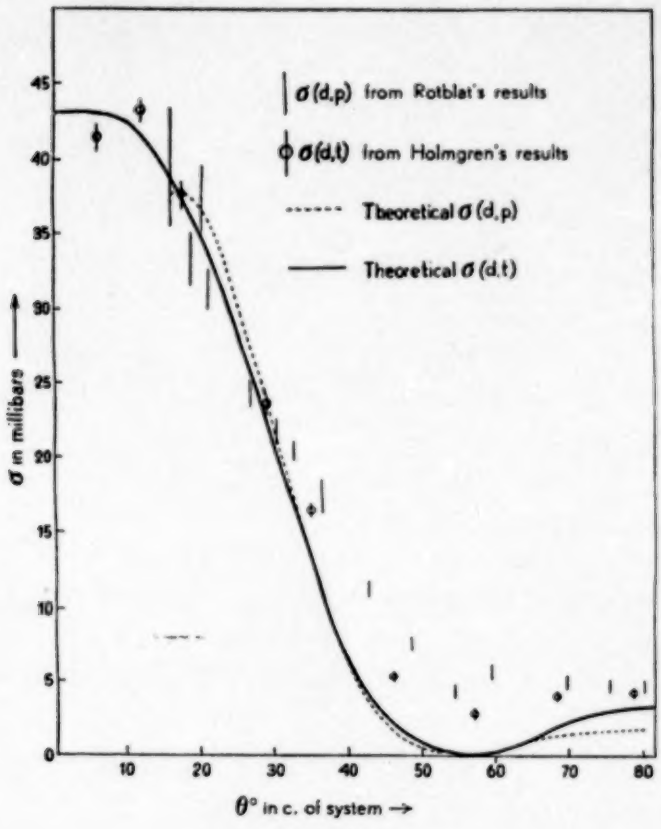

Fig. 2

fig. 2. It is found that the angular dependence of these results is given well by eq. (2) if one takes $l_n = 1$, $r_0 = 5.8 \times 10^{-13}$ cm and $\Phi_T(k)$ to be merely proportional to $(k^2 + \lambda^2)^{-1}$, i.e. proportional to the Fourier transform of the asymptotic form of $f_0(R)$ (eq. (6)). †)

The curve thus obtained is also shown in fig. 2, normalized to the peak value of the experimental curve. Thus, the experiment does not probe into the behaviour of $\Phi_T(k)$ for large k (i.e. the behaviour of $f_0(R)$ for small R). For this, one would need to have (d,t) experiments performed at much higher energies than the one being considered here.

† This can readily be understood from the fact that, for those angles for which a comparison of the experimental and theoretical cross-sections can be made, appropriate values of k^{-2} correspond to distances $\gtrsim 5 \times 10^{-13}$ cm.

In order to obtain information about $|A_0|^2$ we need the results for a (d,p) (or (p,d)) reaction between the same two levels. For this purpose Rotblat's results for the C^{12}(d,p)C^{13} reaction (shown in fig. 2) were chosen. Here the stripping formula (eq. (1)) reproduces the angular distribution quite well, as is also shown in fig. 2, if one again chooses $l_n = 1$, $r_0 = 5.3 \times 10^{-13}$ cm, with $\Phi_d(k)$ the transform of the Hulthen wave function of eq. (7).

The ratio of the preceding experimental cross-sections as a function of angle is given in fig. 3. It can be seen to be practically angle independent over that angular range in which the experimental

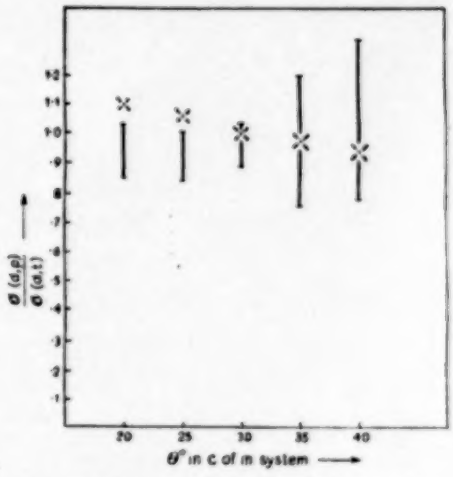

Fig. 3. Values of ratio σ(d,p)/σ(d,t).
I Experimental values with range of error.
× Theoretical values if one takes $C_T^2 |A_0|^2 = 1.8$.

results were considered to be sufficiently accurate to make comparison with theory sensible; the ratio is, in fact, unity within the range of experimental error.

The ratio of the two theoretical curves is also essentially angle independent, and agrees quite well with the experimental ratio if one chooses

$$C_T^2 |A_0|^2 = 1.8. \tag{10}$$

The curve of this ratio as a function of angle is also given in fig. 3. $C_T^2 |A_0|^2$ cannot be in error by more than 8 % at worst.

It is to be noted that this angle independence of σ_1/σ_2 provides a very strong test of the validity of applying simple stripping, since, as is clear from eq. (4), the angle dependence of σ is so complicated as to make any chance agreement of theory and experiment over any finite angular range completely improbable. Thus the agreement

obtained here shows that simple stripping theory can indeed be used for a valid description of the reactions here considered.

From the result (10) one thus finds the upper limit to $|A_0|^2$, using eq. (8) to be

$$|A_0|^2 < 0.11.$$

It is important to note that, since this upper limit is already very small, it is almost certainly a considerable overestimate of the true value, since the use of the resonating group method in obtaining $|C_T|^2$ (section 2) must have resulted in a serious underestimate of it. This value of $|C_T|^2$ would only have been reasonably correct if, on using it, one had indeed found $|A_0|^2$ to be large (of order unity).

In concluding this section, it might be repeated that an actual value of $|A_0|^2$, rather than just an upper limit, could be obtained if experimental results became available for the C^{13}(d,t)C^{12} reaction at considerably higher deuteron energies (say $\gtrsim$ 10 MeV). In this event the angular distribution would determine the form factor Φ_T for sufficiently high momenta so that the normalization constant C_T could be found. (The inaccuracy in the Φ_T thus found for very high momenta — see footnote on page 13 — would have negligible influence on C_T).

4. Discussion

The probability of finding the triton in the configuration of deuteron ground state plus second neutron has been found to be < 0.11, compared to its maximum possible value of 0.75.

Although errors may have arisen due to the neglect of Coulomb and nuclear effects in the simple stripping theory, which would tend to give us too low an answer (since the reduction in the cross-section in the lower energy (d,t) case might be expected to be greater than for the (d,p) case used), yet this effect can be expected to be outweighed by the underestimation of $|C_T|^2$.

Moreover, the good agreement of the theoretical with the experimental cross-section ratios over that angular range within which such comparison was possible, indicates that the simple stripping theory is indeed valid here.

The value of the upper limit to the probability shows that the resonating group method, which deals with the ground configuration as predominant, cannot be used for the three nucleon systems.

Hence calculations based on these methods, such as those of references 6 and 7, for trying to determine the type of nuclear forces actually operating by comparison of the theoretical and experimental cross-sections for (n,d) scattering, must be subject to extremely large errors, which make them of little practical value.

I should like to thank Dr. S. T. Butler for suggesting this problem and for many helpful discussions.

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RENORMALISATION AND MASS LEVELS

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Abstract: The new aspects of the renormalisation problem disclosed by the study of Lee's model are discussed from the point of view of the theory of propagators without making any assumptions about the type of interaction, and introducing from the start a spinor particle with a finite number of stable states of different masses.

1. General Discussion

In a recent paper[1]), Lee has presented an interesting model of interacting particles which is both renormalisable and soluble. The study of this model disclosed a new difficulty in the renormalisation problem: observable coupling constants can be made finite only by starting from non-hermitian interactions, which lead to two mass levels of a particle. In his discussion, Lee applied the ordinary formulation of the renormalisation theory for particles, all of which have one mass level. However, a complete discussion of the renormalisation seems to require extending the usual formulation to cases involving particles with many mass levels. The aim of the present paper is to give such an extension without making any assumptions about the type of interaction and to derive a necessary condition for obtaining unitary S-matrices. As an example the next section will be devoted to a study of the Lee model. The treatment adopted in this paper is based on the theory of propagators [2]), instead of using the Hamiltonian formalism.

We shall consider a case in which a spinor particle has a finite number of different stable states of mass $m^{(1)}, \ldots, m^{(n)}$ $(m^{(1)} < m^{(2)} \ldots < m^{(n)})$. The equations for the one-body-propagator of the spinor particle has the form

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$$(-i\gamma_\mu p_\mu - \varkappa - \mathfrak{M}(-i\gamma p))\, G(p) = 1, \tag{1}$$

with the mechanical mass $\varkappa$ and the mass operator $\mathfrak{M}$.

The mass values of the particle are given by the discrete roots $x = m^{(1)}, \ldots, m^{(n)}$ of the equation[2])

$$x - \varkappa - \mathfrak{M}(x) = 0 \tag{2}$$

in the domain

$$x < C.$$

Here the constant C is defined in such a way that $x \geqq C$ corresponds to states of more than one particle † and that $C > m^{(n)}$ [8]).
We shall asume, for simplicity, that these mass values correspond to simple roots of (2). Thus, we can write the operator

$$h(-i\gamma p) \equiv -i\gamma_\mu p_\mu - \varkappa - \mathfrak{M}(-i\gamma p) \tag{3a}$$

in the form

$$h(-i\gamma p) = a(-i\gamma p) \prod_{j=1}^{n} (-i\gamma_\mu p_\mu - m^{(j)}). \tag{3b}$$

Here $a(x)$ is a function with no discrete zeros for $x < C$.

Corresponding to each mass value $m^{(j)}$ we define a constant $Z^{(j)}$ by

$$\frac{1}{Z^{(j)}} \equiv \frac{\partial}{\partial x}(x - \mathfrak{M}(x))\Big|_{x=m^{(j)}} \tag{4a}$$

$$= a(m^{(j)}) \prod_{k \neq j} (m^{(j)} - m^{(k)}). \tag{4b}$$

Then, it can be shown from (1) that[2])

$$G(p) = \lim_{\varepsilon \to +0} \sum_{(j)} Z^{(j)} \frac{1}{-i\gamma_\mu p_\mu - m^{(j)} + i\varepsilon} \frac{a(m^{(j)})}{a(-i\gamma p)}. \tag{5a}$$

The relation (5a) shows that $Z^{(j)}$ is just the normalisation factor of the propagator of the particle whose mass is $m^{(j)}$. The definition (4a) can be regarded as the extension of Dyson's definition for Z_2 to the case of many mass levels. In order to clarify the physical meaning of the constants $Z^{(j)}$, we shall now calculate the propagator

$$G(x, x') = \frac{1}{(2\pi)^4} \int d^4p\; G(p)\; e^{i(p_\mu x_\mu - x'_\mu)}$$

$$= \lim_{\varepsilon \to +0} \sum_{(j)} \frac{Z^{(j)}}{(2\pi)^4} \int d^4p \frac{i\gamma_\mu p_\mu - m^{(j)}}{p_\mu p_\mu + m^{(j)2} - i\varepsilon} \frac{a(m^{(j)})}{a(-i\gamma p)} \tag{5b}$$

for $t > t'$. Since we take $t > t'$, we can carry out the integration over p_0 in (5b) by taking the path of integration as the half-circle

† When a many-particle state has the energy p_0 and momentum $\mathbf{p}$, x corresponding to this state is equal to

$$x = (p_0^2 - \mathbf{p}^2)^{1/2}.$$

below the real axis. Thus we see that only the poles appearing under the real axes can contribute to (5b), i.e. $p_0 = \sqrt{(\mathbf{p}^2 + m^{(j)2})}$ for all j and, in the domain of $p_0 > \sqrt{(\mathbf{p}^2 + C^2)}$, the zeros of $a(-i\gamma p)$. The pole at $p_0 = \sqrt{(\mathbf{p}^2 + m^{(j)2})}$ contributes to (5b) through an oscillating term with frequency $\sqrt{(\mathbf{p}^2 + m^{(j)2})}$. This contribution can be calculated by using the residue of (5b) at $p_0 = \sqrt{(\mathbf{p}^2 + m^{(j)2})}$ to be

$$-\frac{Z^{(j)}}{(2\pi)^3}\int d\mathbf{p}\,\frac{i\gamma_\mu P_\mu^{(j)} - m^{(j)}}{2P_0^{(j)}}\,e^{i(P_\mu^{(j)},\, x_\mu - x'_\mu)}$$
$$=\frac{Z^{(j)}}{(2\pi)^3}\int d\mathbf{p}\,\frac{P^{(j)} + H^{(j)}}{2P_0^{(j)}}\,\gamma_4\,e^{i(P_\mu^{(j)},\, x_\mu - x'_\mu)} \qquad (5c)$$

with

$$P_\mu^{(j)} = (p,\ i\,P_0^{(j)}) = i\sqrt{(\mathbf{p}^2 + m^{(j)2})}.$$

Here we have used the relation

$$(i\,P_\mu^{(j)}\,\gamma_\mu - m^{(j)})\,(iP_\mu^{(j)}\,\gamma_\mu + m^{(j)})^n = 0,\ (n \text{ integer})$$

and therefore

$$\frac{i\gamma_\mu P_\mu^{(j)} - m^{(j)}}{a(-i\gamma\,P^{(j)})} = \frac{i\gamma_\mu P_\mu^{(j)} - m^{(j)}}{a(-i\gamma\,P^{(j)} - m^{(j)} + m^{(j)})}$$
$$= \frac{1}{a\,(m^{(j)})}\,(i\gamma_\mu\,P_\mu^{(j)} - m^{(j)}).$$

The operator $H^{(j)}$ in (5c) is defined by

$$H^{(j)} = i\gamma_4\,\boldsymbol{\gamma}\cdot\mathbf{p} + m^{(j)}\,\gamma_4 = \boldsymbol{\alpha}\cdot\mathbf{p} + m^{(j)}\,\beta.$$

On the other hand, according to the definition of the Green's function $G(x, x')$, we have

$$\langle\,\text{vac}\,|\,\psi\,(x)\,\psi^*\,(x')\,|\,\text{vac}\,\rangle = G\,(x, x')\,\gamma_4 \qquad \text{for } t > t'.$$

This may be written

$$\sum_{P_\mu}\langle\text{vac}\,|\,\psi\,(x)\,|\,P_\mu\rangle\,\langle P_\mu\,|\,\psi^*\,(x')\,|\,\text{vac}\rangle \qquad \text{for } t > t'.$$

Here P_μ denotes an eigenstate of the total energy-momentum operator and the summation is taken over all such eigenstates. By making use of the Fourier components of $\psi\,(x)$ and $\psi^*(x')$,

$$\psi\,(x) = \sum_{P_\mu}\psi\,(P)\,\frac{1}{\sqrt{V}}\,e^{iP_\mu x_\mu}\ \text{etc.}$$

(where V is the volume) we obtain

$$G\,(x, x')\,\gamma_4$$
$$= \sum_{P_\mu}\frac{1}{V}\langle\text{vac}\,|\,\psi\,(0)\,|\,P_\mu\rangle\,\langle P_\mu\,|\,\psi^*\,(0)\,|\,\text{vac}\rangle\,e^{i(P_\mu,\, x_\mu - x'_\mu)}$$

$$= \frac{1}{(2\pi)^3} \sum_j \int d\mathbf{p} \langle \mathrm{vac} | \psi(0) | P_\mu^{(j)} \rangle \langle P_\mu^{(j)} | \psi^*(0) | \mathrm{vac} \rangle e^{i(P_\mu^{(j)}, x_\mu - x'_\mu)}$$

$$+ \int_C^\infty dp_0 \int d\mathbf{p}\, \varrho\, (\mathbf{p}, p_0) \qquad \text{for } t > t'. \quad (6a)$$

The last term of (6a) denotes the contribution from the eigenstates in the domain $x \geqq C$. Comparing (6a) with (5c), and taking account of the fact that $(P_0^{(j)} + H^{(j)})/2P_0^{(j)} = 1$ for positive energy states, we have

$$Z^{(j)} = \langle \mathrm{vac} \mid \psi(0) \mid P_\mu^{(j)} \rangle \langle P_\mu^{(j)} \mid \psi^*(0) \mid \mathrm{vac} \rangle. \quad (6b)$$

Since the function $a\,(x)$ has no zeros for $x < C$ it has no zeros for $x < m^{(j)}$. Thus, provided it has no singularities in this domain of x, the function a is either positive or negative definite. In the following we will suppose that this condition is satisfied. Then, it follows from (4b) that the sign of $Z^{(j)}$ alternates[2]) with increasing j:

$$Z^{(j)}/Z^{(j+2l+1)} < 0 \ (l \text{ is integer}). \quad (7)$$

In other words, half of the $Z^{(j)}$ are negative, and the others are positive.

Taking into account (6b), we have the curious situation that the matrix element $\langle \mathrm{vac} \mid \psi \mid P_\mu^{(j)} \rangle$ is the complex conjugate of $\langle P_\mu^{(j)} \mid \psi^* \mid \mathrm{vac} \rangle$ for those j only whose $Z^{(j)}$ is positive. Thus, the interaction Hamiltonian $H\,(x)$ containing ψ^* cannot be Hermitian: if the matrix element $\langle a \mid H\,(x) \mid P_\mu^{(j)} \rangle$ ($\mid a \rangle$ denotes an eigenstate of total Hamiltonian) is the complex conjugate of $\langle P_\mu^{(j)} \mid H\,(x) \mid a \rangle$ for $Z^{(j)} > 0$, then it is not so for $Z^{(j)} < 0$ (and vice-versa). Thus, we can expect that the S-matrix is not unitary. The relation (6b) also shows that the expectation value of the number operator for the $m^{(j)}$-state of mass is negative for $Z^{(j)} < 0$, and therefore such a particle appears with a negative probability. This situation gives rise to the following difficulty: Suppose a transition process to occur between $m^{(j)}$ and $m^{(i)}$ states through the creation of a Boson. We shall assume, for simplicity, that the Boson has only one mass value and the renormalisation factor of the vertex Γ is common[††] for all transitions ($i \leftrightarrow j$). Then, the obser-

† The form of the result bears a certain similarity to the corresponding expression in the theory of Pais and Uhlenbeck [4]). We would like to thank Dr. G. N. Fowler for discussions on details of this point.

†† This may not be true in quantum electrodynamics on account of gauge-invariance. In quantum electrodynamics, corresponding to bremsstrahlung by a charged spinor particle in the mass $m^{(i)}$ state, the renormalisation factor $Z_1^{(i)}$ of Γ_μ would be equal to

vable coupling constants $g^{(i,j)}$ corresponding to $(i \leftrightarrow j)$-processes would satisfy relations

$$g^{(i,j)}/g^{(i,j+2l+1)} = (Z^{(j)}/Z^{(j+2l+1)})^{1/2}.$$

These ratios are imaginary on account of (6). When we have processes in which the spinor particle transmutes into another spinor particle through vertex Γ (cf. fig. 2), which is assumed to have common renormalisation factor for all mass states of the spinor particle, the observable coupling constants $g^{(i)}$ satisfy relations

$$g^{(i)}/g^{(j)} = (Z^{(i)}/Z^{(j)})^{1/2}.$$

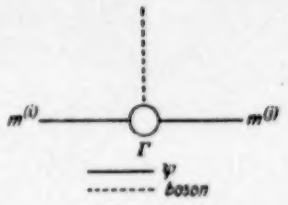

Fig. 1

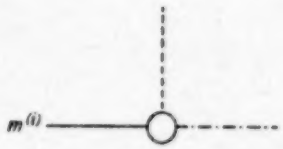

Fig. 2

Here the coupling constant $g^{(i)}$ corresponds to the transmutation process from the $m^{(i)}$-mass state of the spinor particle. Thus, we may find again that some ratios of the observable coupling constants are imaginary, and therefore the S-matrix is not unitary. The method suggested by Lee of taking the unrenormalised coupling constant to be imaginary cannot resolve the difficulty, because the

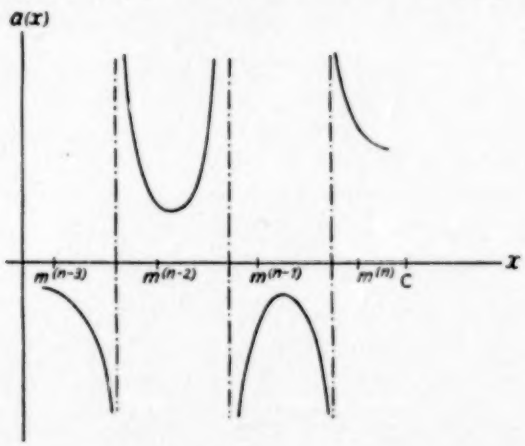

Fig. 3

sign of the ratio (6) is independent of the unrenormalised coupling constant.

$$Z_1^{(i)} = \frac{\partial}{\partial(-i\gamma p)} G(p)\Big|_{i\gamma p = -m^{(i)}} = a(m^{(i)}) \prod_{i \neq k} (m^{(i)} - m^{(k)}) = 1/Z^{(i)}.$$

Thus, it is possible that each factor $Z^{(i)}$ would be cancelled out by $Z_1^{(i)}$. However a non-unitary S-matrix could arise, if Z_3 (the renormalisation factor of the photon propagator) were negative. Moreover, when we take into account non-electromagnetic interactions of the electron, difficulties would appear if $Z^{(i)}$ were negative, because, now, $Z^{(i)}$ can not be cancelled by $Z_1^{(i)}$.

Thus, we may obtain a unitary S-matrix only when all $Z^{(j)}$ are positive. The relation (4b) shows that all $Z^{(j)}$ may be positive only when the function $a(x)$ has a very special distribution of singularities: $a(x)$ must have at least one singular point in each domain $m^{(1)} < x < m^{(2)}$, $m^{(2)} < x < m^{(3)}$, ..., $m^{(n-1)} < x < m^{(n)}$, as fig. 3 shows. This is a necessary condition for making the S-matrix unitary.

2. Lee's Model

We shall apply the general discussion of § 1 to Lee's model [1]. The Hamiltonian in this example is

$$\overline{\mathscr{H}} = \int H d^3x$$

$$H = \varkappa \bar{\psi}_V \psi_V + M \bar{\psi}_N \psi_N + \tfrac{1}{2}\{(\partial\varphi)^2 + \mu^2\varphi^2\} + g(\bar{\psi}_V \psi_N \varphi^+ + \bar{\psi}_N \psi_V \varphi^-).$$

Here $\varkappa$ and M are the mechanical masses of the V- and N-particles, and φ^+ and φ^- are the positive- and negative-frequency parts of the θ-field operator φ. It is clear from the form of the Hamiltonian that the N-particles cannot create θ-particles.

Fig. 4

Thus, the mass operator of the N-particle is zero; the mechanical mass M is just the observable mass. On the other hand, the mass operator $\mathfrak{M}(p_0)$ for the V-particle can be obtained by calculating the process corresponding to fig. 4.

There is no other diagram which contributes to $\mathfrak{M}(p_0)$.

Thus, $\mathfrak{M}(p_0)$ can be given [1] as

$$\mathfrak{M}(p_0) = -\frac{g^2}{(2\pi)^2}\int_0^\infty dk \frac{k^2}{\omega(\omega - p_0 + M)}. \tag{8}$$

Corresponding to (1), we have the equation for the one-V-particle propagator $G(p)$:

$$h(p_0)\, G(p) \equiv (p_0 - \varkappa - \mathfrak{M}(p_0))\, G(p) = 1.$$

According to the discussion in § 1, the stable mass levels can appear for $p_0 < M + \mu$. Then, corresponding to a mass m, the observable coupling constant g_c is given by

$$g_c^2 = g^2 Z.$$

Here the constant Z is defined by

$$\frac{1}{Z} = \frac{\partial}{\partial p_0}(p_0 - \mathfrak{M}(p_0))_{p_0=m} = 1 + \frac{g^2}{(2\pi)^2}\int_0^\infty dk \frac{k^2}{\omega(\omega - m + M)^2}.$$

It is easy to obtain the relation

$$\frac{1}{g_C^2} = \frac{1}{g^2} + \frac{1}{(2\pi)^2}\int_0^\infty dk \frac{k^2}{\omega(\omega - m + M)^2}.$$

This shows that g^2 must be negative in order that g_C be finite. When g^2 is negative, the Hamiltonian cannot be hermitian. Moreover, it will be shown later that $h(p_0)$ has two zeros $p_0 = m_1, m_2$ in the domain $p_0 < M + \mu$:

$$h(m_1) = 0, \tag{9a}$$

$$h(m_2) = 0, \tag{9b}$$

when g^2 is negative. Then, we can obtain two equivalent expressions for $h(p_0)$, viz.

$$h(p_0) = p_0 - m_1 - \mathfrak{M}(p_0) + \mathfrak{M}(m_1)$$

$$= (p_0 - m_1)\left\{1 + \frac{g^2}{(2\pi)^2}\int_0^\infty dk \frac{k^2}{\omega(\omega - p_0 + M)(\omega - m_1 + M)}\right\} \tag{10a}$$

and

$$h(p_0) = (p_0 - m_2 - \mathfrak{M}(p_0) + \mathfrak{M}(m_2))$$

$$= (p_0 - m_2)\left\{1 + \frac{g^2}{(2\pi)^2}\int_0^\infty dk \frac{k^2}{\omega(\omega - p_0 + M)(\omega - m_2 + M)}\right\}. \tag{10b}$$

Here

$$\omega \equiv \sqrt{(k^2 + \mu^2)}.$$

The quantities $\mathfrak{M}(m_1)$ and $\mathfrak{M}(m_2)$ are self-masses for the V-particle in the m_1 and m_2 mass states respectively.

The relations (9a) and (10a) leads to the condition for m_1 and m_2:

$$1 + \frac{g^2}{(2\pi)^2}\int_0^\infty dk \frac{k^2}{\omega(\omega - m_1 + M)(\omega - m_2 + M)} = 0. \tag{11}$$

This is also the condition derived from (9b) and (10b). Thus, we see that there is no reason to give any special importance to one of two mass levels as Lee did†. We can indeed treat them in a symmetrical way. When one of the two mass levels, say m_1, is equal to M, the condition (11) goes over to that obtained by Källen and Pauli††. They derived their condition by solving the eigenvalue problem of the

† Lee discussed the renormalisation constant for one only of the two mass levels.

†† The authors would like to thank Dr. Källen for his kindness in allowing them to read his paper before publication.

total Hamiltonian†. We shall show later that (11) can be satisfied and, therefore, that there exist two mass levels. By using (11) and (10a) (or (10b)), $h(p_0)$ can be written in the form

$$h(p_0) = (p_0 - m_1)(p_0 - m_2)\, a(p_0) \tag{12}$$

with

$$a(p_0) = \frac{g^2}{(2\pi)^2}\int_0^\infty dk \frac{k^2}{\omega(\omega - p_0 + M)(\omega - m_1 + M)(\omega - m_2 + M)}. \tag{13}$$

It is clear from (13) that $a(p_0)$ has no zeros for $p_0 < M + \mu$, where it has a definite sign (the same as that of g^2).

By using (5), $G(p)$ can be written

$$G(p) = Z^{(1)} \frac{1}{p_0 - m_1} \frac{a(m_1)}{a(p_0)} + Z^{(2)} \frac{1}{p_0 - m_2} \frac{a(m_2)}{a(p_0)}. \tag{14}$$

Here, $Z^{(1)}$ and $Z^{(2)}$ are the constants defined by

$$1/Z^{(1)} = a(m_1)(m_1 - m_2) \tag{15a}$$

$$1/Z^{(2)} = a(m_2)(m_2 - m_1) \tag{15b}$$

(cf. (4b)). It is clear from (15a,b) that

$$Z^{(1)}/Z^{(2)} < 0. \tag{16}$$

As was shown by Lee, the vertex Γ is, in the present example, given by

$$\Gamma = 1.$$

Thus, the observable charges g_{c1} and g_{c2}, corresponding to the transitions $(m_1 \to N, \theta)$ and $(m_2 \to N, \theta)$, satisfy the relations

$$g_{c1}^2 = g^2 Z^{(1)}, \tag{17a}$$

$$g_{c2}^2 = g^2 Z^{(2)}. \tag{17b}$$

Their ratio is

$$\frac{g_{c1}}{g_{c2}} = \left(\frac{Z^{(1)}}{Z^{(2)}}\right)^{1/2}.$$

Thus, one of these observable coupling constants must be imaginary.

The relation (13) shows that both of $a(m_1)/g^2$ and $a(m_2)/g^2$ are finite, and therefore $Z^{(1)}/Z^{(2)}$ is finite. Thus, we can take both g_{c1} and g_{c2} to be finite.

† When we compare our results with those obtained by Källen and Pauli, we must take $m_1 = M$.

By using (11), (13) and (15a), we obtain

$$\frac{1}{Z^{(1)}}=(m_1-m_2)\frac{g^2}{(2\pi)^2}\int_0^\infty dk\,\frac{k^2}{\omega(\omega-m_1+M)^2\,(\omega-m_2+M)}$$

$$=\frac{g^2}{(2\pi)^2}\int_0^\infty dk\,\frac{k^2}{\omega(\omega-m_1+M)^2}-\frac{g^2}{(2\pi)^2}\int_0^\infty dk\,\frac{k^2}{\omega(\omega-m_1+M)\,(\omega-m_2+M)}$$

$$=1+\frac{g^2}{(2\pi)^2}\int_0^\infty dk\,\frac{k^2}{\omega(\omega-m_1+M)^2}.$$

This leads to

$$Z^{(1)}=1-\frac{g_{C1}^2}{(2\pi)^2}\int_0^\infty dk\,\frac{k^2}{\omega(\omega-m_1+M)^2}. \tag{18a}$$

In a similar way, we have

$$Z^{(2)}=1-\frac{g_{C2}^2}{(2\pi)^2}\int_0^\infty dk\,\frac{k^2}{\omega(\omega-m_2+M)^2}. \tag{18b}$$

It must be noted that all relations are symmetrical with respect to the two mass levels m_1, m_2. The relation (11) can now be expressed in terms of g_{C1} by

$$1-\frac{g_{C1}^2}{(2\pi)^2}(m_1-m_2)\int_0^\infty dk\,\frac{k^2}{\omega(\omega-m_2+M)\,(\omega-m_1+M)^2}=0$$

on account of (18a) and the relation

$$Z^{(1)}\left(1+\frac{g^2}{(2\pi)^2}\int_0^\infty dk\,\frac{k^2}{\omega(\omega-m_2+M)\,(\omega-m_1+M)}\right)$$

$$=Z^{(1)}+\frac{g_{C1}^2}{(2\pi)^2}\int_0^\infty dk\,\frac{k^2}{\omega(\omega-m_2+M)\,(\omega-m_1+M)}.$$

We can write (11) also in terms of g_{C2}

$$1-\frac{g_{C2}^2}{(2\pi)^2}(m_2-m_1)\int_0^\infty dk\,\frac{k^2}{\omega(\omega-m_1+M)\,(\omega-m_2+M)^2}=0. \tag{19b}$$

It is interesting to note that (19a) and (19b) can also be derived from (17a) and (17b) by using (15a) and (15b).

We shall now consider the condition (19a,b) in the two cases

(i) $g_{C1}^2 > 0$ and therefore $g_{C2}^2 < 0$,

(ii) $g_{C1}^2 < 0$ and therefore $g_{C2}^2 > 0$.

In case (i), (19a,b) leads to $m_2 < m_1$ $(< M + \mu)$. Then (18a,b) shows that $Z^{(1)} < 0$, $Z^{(2)} > 1$ and therefore $g^2 < 0$. By changing $(g_{C1}\, m_1,\, Z^{(1)})$ into $(g_{C2},\, m_2,\, Z^{(2)})$ in this discussion, we obtain the result for case (ii). These results are summarised in table 1. Since g^2 is negative, $a\,(p_0)$ is also negative for $p_0 < (M + \mu)$.

TABLE I

	higher mass	lower mass
Z-factor	< 0	> 1
g_C	real	imaginary
g^2	negative	

In the following discussion, we shall denote the higher and lower mass by m_1 and m_2 respectively.

We shall now restrict the domain of integration over k by $0 < k < K$ $(K \gg \mu)$. Then, the condition (19a) is

$$1 - \frac{g_{C1}^2}{(2\pi)^2}\,(m_1 - m_2)\int_0^K dk\,\frac{k^2}{\omega(\omega - m_2 + M)\,(\omega - m_1 + M)^2} = 0. \quad (20)$$

By using the relations

$$A(m_1, m_2) \equiv (m_1 - m_2)\int_0^K dk\,\frac{k^2}{\omega(\omega - m_2 + M)\,(\omega - m_1 + M)^2} > 0,$$

$$\frac{\partial}{\partial m_1} A(m_1, m_2) < 0,$$

$$\lim_{m_2 \to -\infty} A(m_1, m_2) = \int_0^K dk\,\frac{k^2}{\omega(\omega - m_1 + M)^2},$$

it can be shown that there is a lower limit

$$g_{\text{crit}} = (2\pi)^2 \Big/ \int_0^K dk\,\frac{k^2}{\omega(\omega - m_1 + M)^2} \quad (21)$$

for g_{C1} satisfying (20). This result has been given by Källen and

Pauli [5]) in the case $m_1 = M$. It can be shown from (20), (19b), (18a,b) and (17a,b) that, when the lower mass level goes to $-\infty$ and the higher mass value remains fixed, the observable coupling constants, Z-factors and the unrenormalised coupling constant g behave as shown in Table II:

TABLE II

	higher mass	lower mass
g_c^2	$\to g^2 > 0$	$\to -\infty$
Z-factor	$\Rightarrow -0$	$\to 1$
g^2	$\to -\infty$	

$\to a$ means to decrease to a
$\Rightarrow a$ means to increase to a

When g_{C1} becomes smaller than g_{crit}, $Z^{(1)}$ turns out to be positive on account of (18a), and g^2 becomes also positive; all difficulties disappear with the lower mass levels.

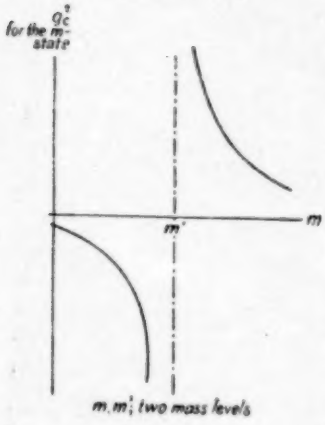

Fig. 5

It is worthwhile noting that, when the lower mass m_2 approaches m_1 instead of going to $-\infty$, g_{C1}^2 and g_{C2}^2 go to $+\infty$ and $-\infty$ respectively, on account of (19a,b) (cf. fig. 5). This is true even when the integration is not cut off at a finite value of k (i.e. $K \to \infty$). As an example of the scattering problem, we shall calculate the phase shift δ for the process $N + \theta \to N' + \theta'$. The momenta of θ and θ' are $\mathbf{k}$ and $\mathbf{k}'$ ($|\mathbf{k}| = |\mathbf{k}'|$) respectively. There is only one diagram (fig. 6) for this process. The S-matrix element is given by

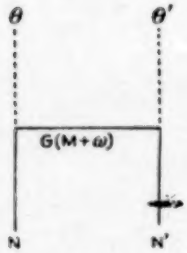

Fig. 6

$$(N', \theta' | S | N, \theta)$$

$$= \lim_{\varepsilon \to 0} \frac{g^2}{2V\omega} G(M + \omega + i\varepsilon)$$

$$= \lim_{\varepsilon \to 0} \frac{g^2}{2V\omega} \sum_{j=1,2} g_{C,j}^2 \frac{a(m_j)}{(\omega + M - m_j + i\varepsilon)\, a(\omega + M + i\varepsilon)} \tag{22}$$

with

$$\omega = \sqrt{(\mathbf{k}^2 + \mu^2)} = \sqrt{(\mathbf{k}'^2 + \mu^2)}$$

(cf. (14)). On the other hand, (15a,b) leads to

$$g_{c1}^2\, a\,(m_1) + g_{c2}^2\, a\,(m_2) = 0. \tag{23}$$

The phase shift δ can be calculated from the relation

$$\tan\delta = \frac{1}{i}\frac{(N',\theta'|S|N,\theta) - (N',\theta'|S|N,\theta)^*}{(N',\theta'|S|N,\theta) + (N',\theta'|S|N,\theta)^*}$$

$$= \frac{\pi|\mathbf{k}|}{(\omega+M-m_1)\,(\omega+M-m_2)}\,\mathrm{P}\int_0^\infty dk \frac{k'^2}{\omega'(\omega'-\omega)(\omega'+M-m_1)\,(\omega'+M-m_2)}. \tag{24}$$

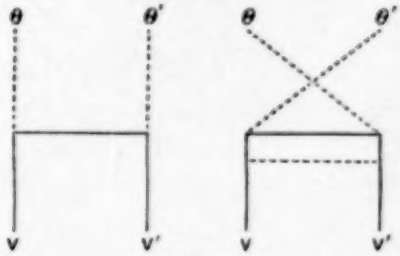

Fig. 7

It is not surprising† that the right hand side of (24) does not depend explicitely on either g_{c1} or g_{c2}, because the latter constants depend uniquely on the mass values m_1 and m_2 on account of (19a,b). There are two diagrams (fig. 7) for the scattering process $V + \theta \to V' + \theta'$. Here V and V' can be in either of the two mass states.

3. Discussion of Lee's Model

The hermitian interaction in Lee's model leads to a mass state of the V-particle whose observable coupling constant is infinite. Thus, if we could have other methods than the renormalisation to resolve the ultra-violet catastrophe, we would not face any difficulty. The use of a non-hermitian interaction can make observable coupling constants finite. However, then, two mass levels appear and the $a(p_0)$-function does not have the property shown by fig. 3; the S-matrix appears to be non-unitary. This is not surprising, because the interaction is not hermitian. It would be interesting to find a model in which hermitian interactions lead to many mass levels, and to see whether the $a(p_0)$-function satisfies the condition shown by fig. 3 or not.

It is interesting to reconsider the renormalisation scheme of quantum electrodynamics in the light of these results. Although there has been no complete calculation of the renormalisation factor

† In fact, we can derive Lee's expression [1]) for $\tan\delta$ from (24) by using (19a). A feature of the expression (24) for $\tan\delta$ is that it is written in terms of mass levels only and that it is symmetric with respect to m_1 and m_2.

Z_3 of the photon propagator, the lowest order approximation of the perturbation calculation has always shown that $Z_2 = -\infty$. Since any value of Z_3 which does not lie between 0 and 1 is incompatible with the physical interpretation of Z_3 as a probability it is a little difficult to understand the successes of the renormalisation procedure in quantum electrodynamics. One possible answer to this question can be obtained from the results of the previous sections. In the high energy domain the effects of various other charged particles may intervene and so we cannot treat the system of electron and electromagnetic field as an isolated system. If we suppose that such effects could lead to a finite mass operator of the electron and photon then there might appear a critical value of the coupling constant such that for values of the electric charge smaller than this critical value the renormalisation constant Z_3 is positive and smaller than 1. Since we have no precise notions about such additional high energy interactions we have no way of checking this suggestion. However, it might be possible to apply such considerations to a simplified model; instead of introducing a cut off in Lee's case, some cohesive field could be introduced to make the mass operator of the V-field finite.

The authors wish to thank Professor Rosenfeld and Dr. G. N. Fowler for their inspiring discussions. One of us wishes to thank Professor N. Bohr for his hospitality at the Institute for Theoretical Physics at Copenhagen.

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FORMULATION MATHÉMATIQUE DU MODÈLE DE GELL-MANN

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Abstract: It is shown that an axiomatic formulation of the Gell-Mann model concerning elementary particles is possible. Different kinds of fields are formally introduced (see table II) which are defined by their transformation properties in ordinary (Lorentz) space and isobaric space †, the latter being three dimensional. The fundamental assumption is that the Lagrangian for strong interactions must be invariant under rotation and reflection in isobaric space. Also postulated are the Lorentz invariance and the conservation of baryons. The most general form of strong interaction Lagrangian that follows from these assumptions is (3.1). From this, the constants of motion I_3 and N are deduced as well as a new constant U. Up to this point no assumption has been made concerning the charges of the quanta of the various fields. The constant U allows for a definition of charge and, if the charges of N and Λ° are postulated one has the general relation

$$Q = I_3 + \frac{U}{2}$$

for all particles. The values of U deduced from the theory turn out to be precisely those postulated in the Gell-Mann model. In fact a complete equivalence with this model is so obtained.

Possibilities of introducing other fields are investigated. It is shown that in most cases some difficulties arise for the definition of U (and therefore of Q). Two different approaches are then possible:

(i) In the Lagrangian constructed according to the above mentioned rules some interaction terms may be suppressed and new fields can so be introduced. This procedure however cannot be achieved without some degree of arbitrariness.

(ii) The following point of view can be adopted: A set of fields is self consistent only when all the true scalars constructed with these fields can, without contradiction, appear in the Lagrangian. This approach is perhaps more satisfactory for, while not being in contradiction with present experimental data, it is at the same time more elegant and may provide a useful means of limiting the number of fields.

1. Introduction

Bien qu'il ne soit pas possible de donner ici un résumé de tous les faits expérimentaux connus concernant les hypérons et les mésons

† Dans cet article, conformément à une récente suggestion, l'expression "spin isobarique" est utilisée au lieu de "spin isotopique".

lourds [1]) un rappel très bref de certains caractères essentiels de ces particules semble indiqué. Les hypérons sont, comme on le sait, habituellement observés grâce à leurs désintégrations. Ainsi ont été identifiés:

$$\begin{aligned}
\Lambda^\circ &\to \mathrm{P} + \pi^- + 37\ \mathrm{MeV} \\
\Sigma^+ &\nearrow \mathrm{P} + \pi^\circ + \approx 115\ \mathrm{MeV} \\
&\searrow \mathrm{N} + \pi^+ + \approx 115\ \mathrm{MeV} \\
\Sigma^- &\to \mathrm{N} + \pi^- + \approx 115\ \mathrm{MeV} \\
\Xi^- &\to \Lambda^\circ + \pi^- + \approx 65\ \mathrm{MeV}.
\end{aligned}$$

Les vies moyennes sont grandes, de l'ordre de 10^{-10} sec; ces particules sont donc métastables par rapport aux désintégrations π et γ. Pendant un certain temps l'idée d'associer le Λ° et les $\Sigma^\pm$, comme différents états de charge de la même particule, a semblé la plus naturelle. Toutefois, cette hypothèse est maintenant abandonnée en raison de l'importante différence des masses. Récemment d'autre part [2]) ont été données des indications en faveur de l'existence d'un Σ° de masse comparable à celle des $\Sigma^\pm$; ce Σ° se désintégrerait rapidement suivant $\Sigma^\circ \to \Lambda^\circ + \gamma$. Le fait que Λ ne possède qu'un état de charge semble un des faits expérimentaux solidement établis.

En ce qui concerne les bosons lourds la situation est moins claire. Différents modes de désintégration de particules chargées et neutres de masse environ 960 m_e ont été observés ($K_{\mu 2}$, $K_{\mu 3}$, $K_{\pi 2}$, $K_{\pi 3}$). Un excès positif important a été mis en évidence [3]). Jusqu'à présent la question de savoir si toutes les particules de même charge sont identiques et susceptibles de modes de désintégration différents ou si au contraire on a affaire à plusieurs particules essentiellement différentes ne semble pas définitivement tranchée.

Enfin un fait expérimental d'une grande importance est le phénomène de la production associée: les particules étranges ne sont apparemment jamais produites isolément. Ceci avait été prédit par certaines théories [4]).

Du point de vue théorique la liaison étroite de toutes les particules les unes avec les autres (prouvée par leurs modes de production et de désintégration) nécessite un traitement d'ensemble. Or cet objectif paraît, vu la complexité des faits concernant les mésons lourds et notre manque partiel d'informations à leur sujet, fort difficile à atteindre d'emblée. Dans ces conditions il semble que le premier objectif d'une théorie doit être de décrire correctement les faits bien établis concernant les hypérons. En raison de la production

associée il n'est toutefois pas possible, même à ce stade, de laisser les mésons lourds entièrement à l'écart. Il est nécessaire d'en introduire au moins un (avec plusieurs états de charge). Nous l'appellerons θ. Dans un stade ultérieur la théorie pourra éventuellement être raffinée sur ce point.

Un des problèmes les plus importants de la théorie des particules élémentaires a toujours été de concilier leur production abondante avec leurs longues vies moyennes. Ce conflit est, comme on le sait, résolu par la considération de la production associée. Il est donc très séduisant de choisir ce phénomène comme fait primordial et de construire la théorie en conséquence.† C'est là le principe des théories que nous allons rappeler brièvement.

Trois types d'interaction sont considérés:

(1) les interactions fortes responsables des forces nucléaires et de la production des hypérons et des mésons lourds;

(2) les interactions électromagnétiques;

(3) les interactions faibles responsables des désintégrations.

D'abord introduite d'une façon purement phénoménologique[4]) cette manière de procéder fut mise sous forme mathématique par Pais[5]) dans sa première tentative. Cette théorie, basée sur un espace isobarique tridimensionnel, ne pouvait malheureusement rendre compte de la métastabilité du Ξ. Dans sa plus récente théorie[6,7]), non sujette à la précédente objection, Pais introduit un espace isobarique quadridimensionnel. Les particules élémentaires sont associées aux représentations irréductibles du groupe de rotation dans cet espace, lesquelles sont caractérisées par deux nombres i^+ et i^- : (i^+, i^-). On admet que pour les hypérons $|\, i^+ - i^- \,| = \frac{1}{2}$, pour les mésons $|\, i^+ - i^- \,| = 0$ ou 1, ce qui permet de définir la charge par les relations

$$q = i_3^+ + i_3^- + \tfrac{1}{2} \qquad \text{pour les fermions lourds,}$$

$$q = i_3^+ + i_3^- \qquad \text{pour les mésons,} \tag{1.1}$$

$$-\, i^\pm \leqq i_3^\pm \leqq i^\pm .$$

Aux particules N, Λ, Σ et Ξ sont attribuées les représentations (i^+, i^-) suivantes: $(\frac{1}{2}, 0)$, $(0, \frac{1}{2})$, $(1, \frac{1}{2})$, $(\frac{1}{2}, 1)$; aux méson π et θ: $(1, 0)$ et $(\frac{1}{2}, \frac{1}{2})$ respectivement.

Toutes les interactions conservent N (nombre de fermions moins

† Une autre méthode consiste en l'introduction de très grands spins (voir réf. [7])

nombre d'antifermions) et la charge. Les interactions fortes conservent i^+, i^- et i_3^+, i_3^-, les électromagnétiques i_3^+, i_3^-, enfin les faibles ne conservent que la charge. De cette façon la métastabilité des particules introduites ci-dessus est bien assurée. Malheureusement avec la définition (1.1) de la charge la théorie prédit en outre l'existence de plusieurs particules métastables telles que Λ^+, Σ^{++} etc., qui n'ont jamais été observées. Nous reviendrons sur ce point au § 5.

Le modèle de Gell-Mann [7, 8, 9]) est essentiellement phénoménologique. L'espace isobarique est ici à trois dimensions, la relation $q(i_3)$ varie de particule à particule, enfin il peut exister des fermions de i entier comme des bosons de i demi-entier. On est conduit alors à postuler pour les diverses particules les propriétés données par la fig. 1 et à supposer que les interactions fortes conservent i, i_3 et la charge, les électromagnétiques i_3 et la charge, les faibles uniquement la charge.

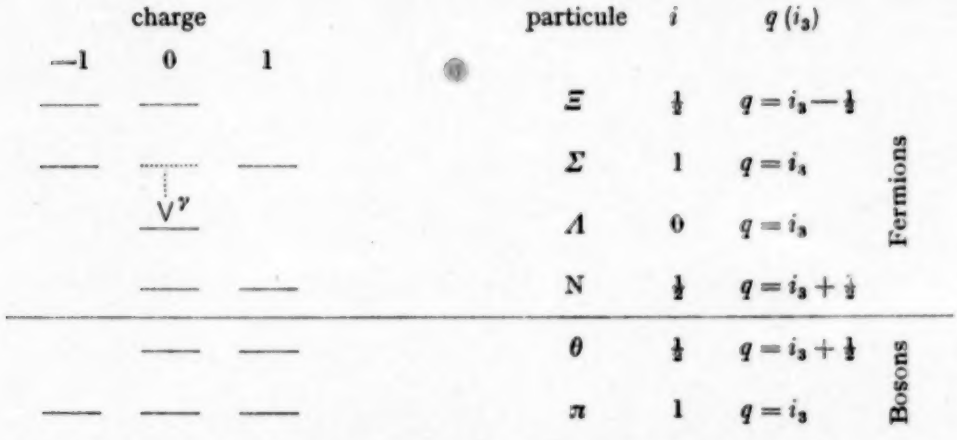

Fig. 1. Les hypérons et les mésons dans la théorie de Gell-Mann

Bien que le nombre de paramètres arbitraires soit élevé le modèle est très utile. Il a permis une classification rationnelle des faits connus et a conduit à la prédiction de certains phénomènes qui paraissent maintenant expérimentalement confirmés. Tels sont, par exemple, l'existence du Σ° (et sa probable instabilité γ), l'impossibilité des réactions du type $N + N \to 2\Lambda$, $\Sigma + \Lambda$, 2Σ, l'excès positif des mésons lourds. Au sujet du dernier point il faut remarquer que les mésons lourds négatifs doivent aussi exister en tant que conjugués de charge (antiparticules) des θ^+ ($\overline{\theta^+} = \theta^-$). De même il existe un $\overline{\theta^\circ}$ différent du θ°. La production d'un $\bar{\theta}$ s'accompagne toujours de celle d'un θ ce qui serait à l'origine de l'excès positif. Il

faut encore ajouter que toutes les désintégrations, jusqu'à présent observées, correspondent dans cette théorie à des transitions avec $\Delta i = \pm \frac{1}{2}$ et $\Delta i_3 = \pm \frac{1}{2}$.

Récemment Sachs [10]) a montré qu'il est possible d'assurer d'une manière phénoménologique la métastabilité des hypérons et mésons lourds ci-dessus considérés sans faire pour cela appel à la notion du spin isobarique mais en introduisant par contre un nouveau nombre quantique, "l'attribut" a. L'attribut est une quantité additive qui pour les N, Λ, Σ, Ξ, π, θ et $\bar{\theta}$ est 0, 1, 1, 2, 0, −1, 1, respectivement. Les interactions fortes conservent l'attribut. Le spin isobarique n'est utilisé que pour définir les différents états de charge. En ce qui concerne les mésons et les hypérons une relation étroite existe entre l'attribut, la composante 3 du spin isobarique, la charge et le nombre de fermions. Nous reviendrons sur ce point au § 4.

En résumé, nous sommes en présence de théories dont l'une au moins, celle de Pais [6]) est mathématiquement satisfaisante mais est difficile à concilier avec les données expérimentales tandis que d'autres, celle de Gell-Mann [7,8]) en particulier, sont plutôt des modèles qui décrivent très correctement les faits observés jusqu'à présent mais souffrent d'un manque de bases théoriques.

Dans ces conditions il a paru intéressant [11]) de rechercher une formulation plus mathématique du modèle de Gell-Mann et de tenter par là de le libérer du reproche d'arbitraire qui lui a été fait.

Le présent travail constitue un essai dans ce sens. Il va presque sans dire que nous sommes loin de considérer notre tentative comme la seule permettant d'atteindre ce but. Il nous semble toutefois que dans l'état actuel des choses, le fait même qu'une solution existe est digne d'être mentionné. Notons enfin qu'une formulation mathématique comme la nôtre est plus rigide que le modèle phénoménologique correspondant, en ce sens qu'elle ne permet pas d'ajouter avec autant de liberté des particules supplémentaires.

L'idée essentielle [11]) de notre méthode est d'introduire, comme Gell-Mann, un espace de spin isobarique à trois dimensions mais à postuler l'invariance du Lagrangien L des interactions fortes par rapport non seulement aux rotations mais aussi aux retournements dans cet espace. L'invariance de Lorentz est postulée ainsi que la conservation du nombre des baryons (suivant Pais nous appelons "baryons" l'ensemble des hypérons et des nucléons). Les champs des particules sont alors décrits d'une manière appropriée par des grandeurs qui sont soit des tenseurs, soit des spineurs dans l'espace

ordinaire et soit des tenseurs soit des spineurs (de 1ère et 2ème espèce) dans l'espace du spin isobarique. Le Langrangien L est ainsi un vrai scalaire ce qui limite beaucoup les possibilités. On est conduit à une définition générale de la charge en fonction de i_3 en ce sens que la connaissance des charges des nucléons et des mésons π permet de déduire les charges des autres particules sans aucune équivoque. Les attributions de Gell-Mann sont ainsi retrouvées, de même que les interactions fortes élémentaires.

Au § 2 quelques propriétés des spineurs dans l'espace à 3 dimensions sont brièvement rappelées. Le § 3 est consacré à la définition et à l'introduction des différénts types de champs utilisés au cours de cet article; les hypothèses concernant les interactions fortes y sont énoncées, le formalisme y est développé et les constantes fondamentales du mouvement y sont trouvées et explicitées. Au § 4 le modèle de Gell-Mann est déduit de la théorie. Une discussion générale est donnée au § 5 et certaines conclusions sont présentées au § 6.

2. Rappel sur les spineurs de 1ère et 2ème espèce

Dans ce paragraphe nous voulons rappeler brièvement quelques propriétés élémentaires des spineurs dans un espace tridimensionnel. Nous suivons ici la méthode géométrique développée par E. Cartan [12,13]).

Considérons un spineur

$$\xi = \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix}$$

dans un tel espace. Soit $\boldsymbol{\alpha}$ un vecteur unitaire réel et P un plan perpendiculaire à $\boldsymbol{\alpha}$. Posons

$$A = \boldsymbol{\alpha} \cdot \boldsymbol{\tau}, \tag{2.1}$$

où les τ_i sont les matrices du spin isobarique. Notons que

$$A^2 = 1, \; A^T = A^*, \tag{2.2}$$

où T signifie "transposé" et * signifie "imaginaire conjugué". A la symétrie par rapport au plan orienté P on attache, par convention, l'opération

$$\xi_A = A\,\xi. \tag{2.3}$$

Il peut être montré que ceci est compatible avec la définition générale des spineurs.

Introduisons la matrice

$$C = i\,\tau_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},\ C^2 = -1, \tag{2.4.}$$

et les spineurs

$$\eta = iC\xi^*. \tag{2.5}$$

Les propriétés de transformation de η lors d'une symétrie se déduisent de celles de ξ. On a

$$\xi_A^* = A^*\,\xi^*,$$

et d'après (2.5)

$$\eta_A = i\,C\,\xi_A^* = i\,C\,A^*\xi^* = -\,CA^*\,C\eta;$$

or
$$CA^*C = A, \tag{2.6}$$

d'où finalement
$$\eta_A = -\,A\eta. \tag{2.7}$$

Il n'est pas possible de trouver une transformation linéaire permettant de passer des ξ aux η. En effet, si B était la matrice d'une telle transformation on devrait avoir simultanément

$$\eta = B\,\xi \text{ et } \eta_A = B\xi_A,$$

ce qui par (2.7) et (2.3) conduit, pour un ξ quelconque, à

$$B\,\xi_A = BA\,\xi = -\,AB\,\xi,$$

donc $BA = -\,AB$ pour tous les A, c'est-à-dire

$$[B, \tau_i]_+ = 0 \text{ pour } i = 1,2,3,$$

ce qui est impossible pour $B \neq 0$.

Les ξ et η sont donc essentiellement différents. On appelle les ξ et η spineurs de 1ère et de 2ème espèce respectivement. Au contraire ξ^* et η (et de même ξ et η^*) sont évidemment des spineurs de même espèce; ils se déduisent l'un de l'autre par la transformation (2.5) qui représente une rotation de 180° autour de l'axe Oy.

Soient ξ, ξ' et η, η' des spineurs de première et deuxième espèce respectivement. Il est bien connu que $\xi'^{*T}\xi$ est un vrai scalaire. A l'aide de (2.3) et (2.2) on vérifie immédiatement son invariance par rapport à toute symétrie, donc aussi par rapport à toute rotation. De même $\eta'^{T}C\xi$ est un vrai scalaire ce que l'on vérifie par (2.3), (2.7) et (2.6). Par contre les quantités $\xi'^{T}\xi$, $\eta'^{*T}C\xi$ etc. ne sont pas des scalaires.

On peut remarquer que les scalaires construits avec des spineurs de même espèce contiennent un * tandis que ceux construits avec

des spineurs d'espèce différentes contiennent deux * ou pas de *. Les différentes grandeurs tensorielles formées à l'aide des spineurs sont données par le tableau I.

TABLEAU I

Les différentes grandeurs tensorielles formées à l'aide de spineurs de première et deuxième espèce ξ et η. Ajouter les hermitiques et les imaginaires conjuguées.

scalaire	$\xi'^{*T}\xi$	$\eta'^{*T}\eta$	$\eta'^{T}C\xi$
pseudo-vecteur	$\xi'^{*T}\tau\xi$	$\eta'^{*T}\tau\eta$	$\eta'^{T}C\tau\xi$
vecteur	$\eta'^{*T}\tau\xi$	$\eta'^{T}C\tau\eta$	$\xi'^{T}C\tau\xi$
pseudo-scalaire	$\eta'^{*T}\xi$	$\eta'^{T}C\eta$	$\xi'^{T}C\xi$

3. Les champs et leurs interactions

a. Introduction des différents types de champs

Les différents champs possibles seront définis à l'aide de leurs propriétés de transformations aussi bien dans l'espace de Lorentz que dans l'espace tridimensionnel du spin isobarique. Lorsqu'il s'agira de ces dernières nous utiliserons les mots "isospineurs" et „isotenseurs". De ce point de vue une particule sera définie en lui attribuant des caractères tensoriel et isotensoriel donnés. Aucune corrélation n'existe entre ces deux attributs. On peut donc avoir des particules spineur - isospineur, spineur - isotenseur, tenseur - isotenseur et tenseur - isospineur; une classification plus poussée en isospineurs de 1ère et 2ème espèce sera d'ailleurs nécessaire. Nous nous limiterons à des particules dont le spin et l'isospin sont inférieurs ou égaux à 1. Tous les hypérons sont considérés ici comme des particules de spin $\frac{1}{2}$ décrites par des spineurs, les mésons comme des bosons de spin zéro ou un. Ces restrictions n'ont cependant rien d'essentiel.

Aucune distinction n'est faite entre les différentes espèces de spineurs (espace ordinaire). Dans une théorie plus élaborée il sera toujours possible de pousser l'analyse sur ce point. Les hypérons sont donc différenciés à l'aide des différents caractères isotensoriels ou isospinoriels qui leurs sont attribués. De même pour les mésons.

Ne voulant pas préjuger pour l'instant de la nature des particules que nous introduisons ainsi nous dénotons les champs correspondants par des symboles peu évocateurs ψ_A, ψ_B, φ_Δ... Le tableau II indique les notations.

TABLEAU II

Définitions des différents champs. Les ψ dénotent des spineurs, les φ des tenseurs, les indices en capitales correspondent à des isospineurs, les indices en minuscules à des isotenseurs.

Champs	Espace de Lorentz	Espace du spin isobarique
ψ_A	spineur	isospineur 1ère espèce
ψ_B	,,	isospineur 2ème espèce
ψ_a	,,	spin zéro(iso-scalaire)
ψ_b	,,	spin 1(iso-ps. vecteur)
φ_Δ	tenseur	isospineur 1ère espèce
φ_δ	spin zéro	spin un(iso-ps. vecteur)

Cette liste n'est bien entendu pas limitative. On pourrait ainsi définir des champs $\varphi_{\Delta'}$ (tenseur, isospineur 2ème espèce) φ_γ (tenseur, isoscalaire), $\psi_{a'}$ (spineur, iso-pseudoscalaire), $\psi_{b'}$ (spineur, isovecteur) etc. Toutefois l'introduction de tels champs en sus de ceux du tableau II ou bien n'apporte rien de nouveau ou bien soulève certains problèmes d'interprétation que nous préférons remettre au § 5.

Dans le présent paragraphe et les suivants nous nous limiterons donc à l'étude des champs donnés par le tableau II.

b. Interactions fortes

Comme dans toutes les théories l'invariance du Lagrangien par rapport aux transformations de Lorentz est postulée ainsi que la conservation des baryons, c'est à dire que

N = nombre de baryons moins nombre d'antibaryons

est une constante du mouvement. Comme dans la théorie de Gell-Mann on suppose l'invariance du Lagrangien L des interactions fortes par rapport aux rotations dans l'espace du spin isobarique. De plus, et c'est là le postulat particulier à la présente théorie, on impose l'invariance de ce Lagrangien par rapport aux symétries dans cet espace.

L'introduction d'isospineurs de 1ère et 2ème espèce dans le tableau II est dès lors rendue nécessaire (voir § 2). La conservation de N exige que les termes de L contiennent le produit d'un ψ^* par un ψ. L'invariance de L dans les rotations et retournements dans l'espace du spin isobarique conduit à ne conserver que les vrais scalaires dans cet espace. Le nombre de termes possibles dans le Lagrangien des interactions fortes est ainsi sévèrement limité †. Au sujet de leur

† *Note sur épreuves:* Il est nécessaire de se restreindre explicitement à des interactions de Yukawa. (A. Messiah, communication privée.)

construction explicite indiquons qu'il suffit d'utiliser les matrices 1 et τ prises entre deux isospineurs (donc pas nécessairement entre deux fermions)†. Par ailleurs on se restreindra à une théorie linéaire. Enfin les termes contenant les dérivées des champs seront négligés afin de ne pas alourdir l'exposé; leur introduction ne modifierait pas les résultats.

De cette façon des termes du type†† $\tilde{\psi}_A^{*T}\tau\psi_A\varphi_\delta$, $\tilde{\psi}_a^{*}\varphi_\Delta^{*T}\cdot 1\cdot\psi_A$, $\tilde{\psi}_b^{*}\varphi_\Delta^{*T}\tau\psi_A$ aux satisfont conditions imposées tandis que par exemple $\tilde{\psi}_a^{*}\varphi_\Delta^{T}\cdot 1\cdot\psi_A$ etc. sont à rejeter par les arguments présentés au § 2 (pour les isoscalaires possibles voir le tableau I).

Utilisant ces propriétés il n'est pas difficile d'écrire la densité $\mathscr{L}$ du Lagrangien de toutes les interactions fortes possibles entre les champs introduits dans le tableau II. Le résultat est

$$\begin{aligned}\mathscr{L} = {} & \tilde{\psi}_A^{*T}\beta\tau\psi_A\varphi_\delta + \tilde{\psi}_a^{*}\beta\psi_b\varphi_\delta + \tilde{\psi}_b^{*}\beta\psi_a\varphi_\delta + [\tilde{\psi}_b^{*}\beta\wedge\psi_b]\varphi_\delta + \\ & + \psi_B^{*T}\beta\tau\psi_B\varphi_\delta + \psi_a^{*}\beta\varphi_\Delta^{*T}\psi_A + i\,\tilde{\psi}_a^{*}\beta\psi_B^{T}C\varphi_\Delta + \\ & + \tilde{\psi}_b^{*}\beta\varphi_\Delta^{*T}\tau\psi_A + i\tilde{\psi}_b^{*}\beta\psi_B^{T}C\tau\varphi_\Delta + \text{c.c.} \qquad (3.1)\end{aligned}$$

où C est la matrice (2.4). Le tableau I indique que (3.1) est un vrai isoscalaire.

Dans (3.1) on a omis les constantes de couplage. Il est supposé qu'elles sont toutes de l'ordre de grandeur de celle des forces nucléaires. Enfin, pour tenir compte du caractère tensoriel de chaque champ mésique il conviendrait d'introduire dans (3.1) les matrices γ appropriées.

c. Développements dans l'espace des moments

Pour les champs spineur-isospineur et tenseur-isotenseur ces développements sont bien connus. Il est peut être opportun de dire ici quelques mots au sujet de ceux qui concernent les bosons isospineurs (φ_Δ) et les fermions isotenseurs (ψ_a et ψ_b).

Soit $\varphi_\Delta = \begin{pmatrix}\varphi_{\Delta_1}\\ \varphi_{\Delta_2}\end{pmatrix}$, φ_{Δ_1} et φ_{Δ_2} étant complexes. Dans les notations

† Une méthode équivalente consisterait à n'introduire formellement des matrices de spin isobarique qu'entre les fermions. Ces matrices seraient en général rectangles.

†† Notations: $\sim$ transposition des composantes spinorielles, T transposition des composantes isosponorielles, $*$ conjugaison de toutes les composantes. Ainsi, ψ étant un spineur-isospineur donc une matrice à 8 lignes et 1 colonne, $\tilde{\psi}$ possède 2 lignes et 4 colonnes, $\tilde{\psi}^T$ 4 lignes et 2 colonnes, $\tilde{\psi}^T$ 1 ligne et 8 colonnes.

de Wentzel[14]) on a

$$\varphi_{\Delta_i} = \sum_k \frac{1}{\sqrt{2\omega_k}} [a_k(\Delta_i) + b_k^*(\Delta_i)] e^{ikx}$$

$$\varphi_{\Delta_i}^* = \sum_k \frac{1}{\sqrt{2\omega_k}} [a_k^*(\Delta_i) + b_k(\Delta_i)] e^{-ikx} \qquad (i = 1,2). \qquad (3.2)$$

Ici $a_k(\Delta_i)$, $a_k^*(\Delta_i)$ sont les opérateurs d'absorption et d'émission respectivement d'une particule Δ_i; de même $b_k(\Delta_i)$ et $b_k^*(\Delta_i)$ pour une particule $\bar{\Delta}_i$. Ces opérateurs obéissent aux règles de commutation habituelles. La densité du Lagrangien du champ libre est

$$\mathscr{L}_\Delta = -\mu_\Delta^2 \varphi_\Delta^{*T} \varphi_\Delta - \frac{\partial \varphi_\Delta^{*T}}{\partial x_\nu} \frac{\partial \varphi_\Delta}{\partial x_\nu}$$

Appliquant à $\mathscr{L}_\Delta$ une rotation infinitésimale dans l'espace du spin isotopique on obtient, pour le spin isotopique $\mathbf{I}_\Delta$ par la méthode usuelle[15])

$$\mathbf{I}_\Delta = -\frac{i}{2} \int [\pi_\Delta^T \boldsymbol{\tau} \varphi_\Delta - \varphi_\Delta^{*T} \boldsymbol{\tau} \pi_\Delta^*] \, dv, \qquad (3.3)$$

où π_Δ est le moment conjugué de φ_Δ. Introduisant dans (3.3) (3.2) et les développements correspondants pour π_Δ on obtient pour $I_{\Delta 3}$

$$I_{\Delta 3} = \tfrac{1}{2} \Sigma_k [n_k(\Delta_1) - n_k(\bar{\Delta}_1) - n_k(\Delta_2) + n_k(\bar{\Delta}_2)], \qquad (3.4)$$

où

$$n_k(\Delta_i) = a_k^*(\Delta_i) a_k(\Delta_i); \; n_k(\bar{\Delta}_i) = b_k^*(\Delta_i) b_k(\Delta_i) \qquad (3.5)$$

représentent les nombres de particules Δ_i et $\bar{\Delta}_i$ respectivement sur l'onde k. (3.4) indique que les i_3 des particules Δ_1, $\bar{\Delta}_1$, Δ_2, $\bar{\Delta}_2$, sont $\frac{1}{2}$, $-\frac{1}{2}$, $-\frac{1}{2}$, $\frac{1}{2}$ respectivement.

Pour ψ_a aucun problème ne se pose. Ce spineur-isoscalaire décrit une particule a et son antiparticule $\bar{a}$. Reste à interpréter ψ_b. La densité du Lagrangien du champ libre est

$$\mathscr{L}_b = -\frac{1}{i} [\tilde{\psi}_b^* \beta \gamma_\mu \frac{\partial \psi_b}{\partial x_\mu} + \mu_b \tilde{\psi}_b^* \beta \psi_b], \qquad (3.6)$$

d'où l'on déduit, comme pour (3.3),

$$\mathbf{I}_b = \frac{1}{i} \int [\tilde{\psi}_b^* \wedge \psi_b] \, dv. \qquad (3.7)$$

En particulier

$$I_{b3} = \frac{1}{i} \int [\tilde{\psi}_{b_1}^* \psi_{b_2} - \tilde{\psi}_{b_2}^* \psi_{b_1}] \, dv. \qquad (3.8)$$

Par la transformation

$$\begin{aligned}
\psi_{b_1} &= \frac{1}{\sqrt{2}}(\psi_{b'} + \psi_{b''}); \quad \psi^*_{b_1} = \frac{1}{\sqrt{2}}(\psi^*_{b'} + \psi^*_{b''}) \\
\psi_{b_2} &= \frac{1}{i\sqrt{2}}(\psi_{b''} - \psi_{b'}); \quad \psi^*_{b_2} = \frac{1}{i\sqrt{2}}(\psi^*_{b''} - \psi^*_{b'}) \\
\psi_{b_3} &= \psi_{b'''} \qquad\qquad ; \quad \psi^*_{b_3} = \psi^*_{b'''}
\end{aligned} \tag{3.9}$$

et par un développement analogue à (3.2), (3.8) devient

$$\begin{aligned}
I_{b3} &= \int [\tilde{\psi}^*_{b'} \psi_{b'} - \tilde{\psi}^*_{b''} \psi_{b''}]\, dv \\
&= \Sigma_k [n_k(b') - n_k(\overline{b'}) - n_k(b'') + n_k(\overline{b''})].
\end{aligned} \tag{3.10}$$

Il est légitime d'introduire les particules b', b'', b''' et leurs antiparticules $\overline{b'}$, $\overline{b''}$, $\overline{b'''}$, dont les i_3 sont d'après (3.10) 1, -1, 0 et -1, 1, 0, comme des quanta indépendants. Substituant (3,9) dans (3.6) on obtient en effet

$$\mathscr{L}_b = -\frac{1}{i}\left[(\tilde{\psi}^*_{b'} \beta\gamma_\mu \frac{\partial \psi_{b'}}{\partial x_\mu} + \mu_b \tilde{\psi}^*_{b'} \beta \psi_{b'}) + ('') + (''')\right]$$

qui est une somme de trois termes de même forme.

En ce qui concerne les champs ψ_A et ψ_B il va de soi qu'ils représentent les particules A_1, A_2 et B_1, B_2 ainsi que leurs antiparticules $\overline{A}_1$, $\overline{A}_2$ et $\overline{B}_1$, $\overline{B}_2$ tandis que φ_δ représente un champ mésique avec trois états de charge δ', δ'', δ'''. Notons qu'il ne serait pas opportun d'employer ici les mots "conjugués de charge" étant donné que les charges des différentes particules introduites n'ont pas encore été précisées.

d. Recherche des constantes du mouvement

Le premier terme de (3.1) est (à une matrice γ_5 près, voir plus haut) la densité du Lagrangien des interactions nucléaires. Ceci conduit à identifier φ_δ avec le champ des mésons π ; φ_δ est donc une grandeur réelle. De même ψ_A est identifié avec le champ des nucléons. Les deux composantes de cet isospineur, ψ_{A_1} et ψ_{A_2} représentent donc respectivement les protons et les neutrons. Les charges q des quanta des autres champs et les relations entre q et i_3, seront déduites de la théorie.

Nous en venons maintenant au point qui nous semble essentiel dans nos considérations. La densité du Lagrangien $\mathscr{L}$ a été construite de telle manière que les opérateurs $\mathbf{I}^2$ et I_3 (où $\mathbf{I}$ est le spin isobarique

total) sont des constantes du mouvement. Il est facile de construire explicitement ces opérateurs (voir § 3*c*) qui, bien entendu, commutent avec l'Hamiltonien total H. De même on peut construire l'opérateur N (conservation des baryons), qui lui aussi commute avec H.

Ces constantes sont les seules qui aient été introduites dès le départ. Toutefois il est important de remarquer que, avec le choix des champs du tableau II, ces constantes ne sont pas les seules. *Il en existe une autre, tout à fait indépendante, qui sera appelée U.*

Pour démontrer ceci il suffit de trouver une transformation des champs qui laisse le Lagrangien invariant et qui ne se déduise pas de celles qui correspondent aux constantes précédentes. Or on s'aperçoit bien aisément, à l'aide de (3.1), que la transformation† (α = constante)

$$\begin{aligned} &\psi_A \to \psi_A e^{i\alpha} && \psi_B \to \psi_B e^{-i\alpha} \\ &\psi_a \to \psi_a && \psi_b \to \psi_b \\ &\varphi_\Delta \to \varphi_\Delta e^{i\alpha} && \varphi_\delta \to \varphi_\delta \end{aligned} \tag{3.11}$$

possède ces propriétés.

De la manière habituelle [15]) on déduit de (3.11) et du Lagrangien des champs libres

$$U = \int [\tilde{\psi}_A^{*T} \psi_A - \tilde{\psi}_B^{*T} \psi_B - i\,(\pi_\Delta^T \varphi_\Delta - \pi_\Delta^{*T} \varphi_\Delta^*)]\, dv. \tag{3.12}$$

Étant donné que I_3, N, U et H commutent entre eux ils peuvent être simultanément diagonalisés. Dans la représentation où l'opérateur nombre de particules est diagonal on obtient ($A \equiv \mathrm{N}$, $\delta \equiv \pi$)

$$\begin{aligned} 2I_3 = {} & n(\mathrm{P}) - n(\overline{\mathrm{P}}) - n(\mathrm{N}) + n(\overline{\mathrm{N}}) + n(B_1) - n(\overline{B_1}) - n(B_2) \\ & + n(\overline{B_2}) + 2[n(b') - n(\overline{b'}) - n(b'') + n(\overline{b''})] + n(\Delta_1) \\ & - n(\overline{\Delta_1}) - n(\Delta_2) + n(\overline{\Delta_2}) + 2[n(\pi^+) - n(\pi^-)], \end{aligned} \tag{3.13}$$

$$\begin{aligned} N = {} & n(\mathrm{P}) - n(\overline{\mathrm{P}}) + n(\mathrm{N}) - n(\overline{\mathrm{N}}) + n(B_1) - n(\overline{B_1}) + n(B_2) \\ & - n(\overline{B_2}) + n(a) - n(\bar{a}) + n(b') - n(\overline{b'}) + n(b'') - n(\overline{b''}) \\ & + n(b''') - n(\overline{b'''}), \end{aligned} \tag{3.14}$$

$$\begin{aligned} U = {} & n(\mathrm{P}) - n(\overline{\mathrm{P}}) + n(\mathrm{N}) - n(\overline{\mathrm{N}}) - n(B_1) + n(\overline{B_1}) - n(B_2) \\ & + n(\overline{B_2}) + n(\Delta_1) - n(\overline{\Delta_1}) + n(\Delta_2) - n(\overline{\Delta_2}), \end{aligned} \tag{3.15}$$

avec $n = \Sigma_k n_k$.

On constate que U est bien une nouvelle constante et non une combinaison linéaire des précédentes.

† $\psi_A \to \psi_A e^{i\alpha}$ signifie $\psi_{A_1} \to \psi_{A_1} e^{i\alpha}$, $\psi_{A_2} \to \psi_{A_2} e^{i\alpha}$. De même pour les autres champs.

4. Déduction du modèle de Gell-Mann

Dans le paragraphe précédent les constantes du mouvement I_3, N et U ont été introduites. Il est intéressant de chercher si une combinaison linéaire de ces quantités ne se prête pas plus aisément à l'interprétation physique de la théorie. A cet effet formons

$$Q = I_3 + \frac{U}{2}. \tag{4.1}$$

A l'aide des formules (3.13) et (3.15) on a

$$\begin{aligned} Q = {} & n(\mathrm{P}) - n(\overline{\mathrm{P}}) - n(B_2) + n(\overline{B_2}) + n(b') - n(\overline{b'}) \\ & - n(b'') + n(\overline{b''}) + n(\Delta_1) - n(\overline{\Delta_1}) + n(\pi^+) - n(\pi^-). \end{aligned} \tag{4.2}$$

En ce qui concerne les nucléons et les mésons π on constate que cette expression n'est rien d'autre que la charge (en unité e) des particules. Il est donc naturel de considérer (4.1) comme la définition générale de la charge d'un système de particules. A titre d'exemple, la formule (4.2) donne pour les particules B_1, $\overline{B_1}$, B_2, $\overline{B_2}$ les charges 0, 0, -1, $+1$ respectivement; pour U (3.15) donne -1, $+1$, -1 $+1$. La relation entre I_3 et Q, Q (I_3) est donc

$Q = I_3 - \frac{1}{2}$ pour une particule B_1 ou B_2,
$Q = I_3 + \frac{1}{2}$ pour une antiparticule $\overline{B_1}$ ou $\overline{B_2}$.

L'ensemble de ces résultats est représenté sous la forme du tableau III.

TABLEAU III

Caractéristiques des différentes particules et leurs attributions. Pour les antiparticules changer les signes de I_3, U et donc Q, et dans la relation $Q(I_3)$, changer le signe de la constante étant donné que la relation (4.1) est généralement valable.

Particules	I	I_3	U	Q	$Q(I_3)$	Attributions
A_1, A_2	$\frac{1}{2}$	$\frac{1}{2}$, $-\frac{1}{2}$	1	1, 0	$I_3 + \frac{1}{2}$	P, N
a	0	0	0	0	I_3	Λ°
b', b''', b''	1	1, 0, -1	0	1, 0, -1	I_3	Σ^+, Σ°, Σ^-
B_1, B_2	$\frac{1}{2}$	$\frac{1}{2}$, $-\frac{1}{2}$	-1	0, -1	$I_3 - \frac{1}{2}$	Ξ°, Ξ^-
δ', δ''', δ''	1	1, 0, -1	0	1, 0, -1	I_3	π^+, π°, π^-
Δ_1, Δ_2	$\frac{1}{2}$	$\frac{1}{2}$, $-\frac{1}{2}$	1	1, 0	$I_3 + \frac{1}{2}$	θ^+, θ°

La dernière colonne du tableau III montre qu'il est possible d'assimiler les quanta de chacun des champs ψ_A, ψ_B etc. introduits ci-dessus avec une particule du modèle de Gell-Mann et que les

valeurs des charges sont correctement retrouvées. Les relations $Q(I_3)$ données par l'avant-dernière colonne sont précisément celles qui sont postulées dans le modèle en question.

D'autre part le Lagrangien (3.1) avec les attributions du tableau III donne les transitions fortes virtuelles entre les différentes particules. On obtient ainsi $\Lambda + \theta^+ \rightleftarrows \mathrm{P}$, $\Sigma^- + \theta^+ \rightleftarrows \mathrm{N}$, $\Xi^- + \theta^+ \rightleftarrows \Lambda$, $\Lambda \rightleftarrows \mathrm{P} + \overline{\theta^+}$ etc., réactions qui, comme on le vérifie immédiatement, conduisent à toutes les transitions réelles fortes prévues par le modèle de Gell-Mann. *Ainsi il y a équivalence complète entre ce dernier et le présent formalisme, à cette différence près que la valeur de U est ici déduite de la théorie et non pas postulée pour chaque particule individuellement.*

Une remarque au sujet de la définition (4.1) de la charge s'impose ici. S'il est vrai qu'à priori — nulle référence n'étant faite à l'expérience — n'importe quelle autre définition de la charge est concevable, en particulier la relation linéaire

$$Q' = \alpha I_3 + \beta N + \gamma U,$$

il est cependant évident que les charges observées des nucléons imposent déjà $\alpha = 1$ et, d'après (3.14) et (3.15), $\beta + \gamma = \frac{1}{2}$. En outre la nécessité d'avoir des charges entières pour toutes les particules donne pour β un nombre entier; la valeur $\beta = 0$ correspond à (4.1). Pour $\beta \neq 0$ un quantum du champ ψ_a serait chargé bien que son spin isobarique soit nul, ce qui ne semble guère attrayant.

Il est possible d'obtenir la définition (4.1) sans exiger que $Q = 0$ pour $I = 0$, argument qui est peut-être sujet à certaines critiques. Pour ceci il suffit de faire dès le début les attributions $\psi_\Lambda \rightarrow$ nucléons et $\psi_a \rightarrow$ particule Λ°, et celles-là seulement. La charge zéro du Λ° conduit alors à $\beta = 0$ et toutes les autres charges, y compris celles des mésons π, sont déduites de la théorie. (4.1) est ainsi obtenu à l'aide du nombre minimum de données expérimentales.

En ce qui concerne l'opérateur U défini par (3.12) remarquons qu'une interprétation simple peut lui être donnée: (3.12) ne contient que des champs de spin isobarique $\frac{1}{2}$ et si l'on convient d'appeler „isofermions" les particules N, θ et $\overline{\Xi}$ et „antisofermions" les $\overline{\mathrm{N}}$, $\overline{\theta}$ et Ξ, U s'écrit [11])

$$U = \text{nombre d'isofermions moins nombre d'antisofermions} \quad (4.3)$$

comme on le voit sur (3.15). Les autres particules sont des „isobosons".

A la loi de conservation des fermions (espace ordinaire) doit

ainsi s'ajouter la loi de conservation des "isofermions" (espace du spin isobarique).
Les règles de Gell-Mann peuvent donc être énoncées de la manière suivante: Les interactions fortes (et électromagnétiques) conservent

(1) la charge,
(2) le nombre des fermions (N constant),
(3) le nombre des isofermions (U constant).

A titre d'exemple donnons des réactions permises:

$$\pi^- + \underset{\text{isof.}}{\mathrm{P}} \to \underset{\text{isof.}}{\Lambda^\circ} + \theta^\circ, \qquad \pi^- + \underset{\text{isof.}}{\mathrm{P}} \to \underset{\text{anti isof.}}{\Xi^-} + \underset{\text{isof.}}{\theta^+} + \underset{\text{isof.}}{\theta^\circ}$$

$$\underset{\text{isof.}}{\mathrm{N}} + \underset{\text{isof.}}{\mathrm{P}} \to \underset{\text{isof.}}{\Sigma^+} + \underset{\text{isof.}}{\theta^\circ} + \underset{\text{isof.}}{\mathrm{N}},$$

et des interdites:

$$\underset{\text{isof.}}{\mathrm{N}} + \underset{\text{isof.}}{\mathrm{N}} \to \underset{\text{isof.}}{\Lambda^\circ} + \underset{\text{isof.}}{\mathrm{N}}, \qquad \pi + \underset{\text{isof.}}{\mathrm{N}} \to \underset{\text{isof.}}{\theta} + \underset{\text{isof.}}{\mathrm{N}} \quad \text{ou } \Sigma + \pi,$$

$$\underset{\text{isof.}}{\mathrm{N}} + \underset{\text{isof.}}{\mathrm{N}} \to \Lambda + \Lambda \text{ ou } \Sigma + \Sigma \text{ ou } \Sigma + \Lambda \text{ ou } \underset{\text{anti isof.}}{\Xi} + \Lambda.$$

Signalons enfin que les nombres quantiques utilisés par certains auteurs sont des combinaisons linéaires de N et U. Ainsi l'attribut de Sachs (voir l'introduction) est $a = N - U$ et l'étrangeté de Gell-Mann est $s = U - N = -a$.

5. Extension possible et remarques diverses

Lors de l'introduction des différents types de champs (§ 3*a*) il a été signalé que la liste des champs contenus dans le tableau II — et qui conduit au modèle de Gell-Mann — n'est pas limitative même si l'on se restreint à des spins isobariques inférieurs ou égaux à 1. C'est ainsi par exemple que la possibilité de définir un champ $\psi_{a'}$ (spineur-isopseudoscalaire) a été mentionnée. Ajoutons aux champs du tableau II le champ $\psi_{a'}$ et complétons la densité de Lagrangien $\mathscr{L}$ (3.1) en écrivant tous les vrais scalaires faisant intervenir $\psi_{a'}$. Avec les attributions du tableau III on obtient

$$\tilde{\psi}^*_{a'} \beta \varphi^T_\theta C \psi_N + \tilde{\psi}^*_{a'} \beta \varphi^{*T}_\theta \psi_\Xi + \text{c.c.} \tag{5.1}$$

La généralisation de la transformation (3.11) conduisant à la constante du mouvement U se heurte alors à une difficulté. En effet le premier terme de (5.1), compte tenu de (3.11), exige $\psi_{a'} \to \psi_{a'} e^{2i\alpha}$ tandis que le second terme de (5.1) exige $\psi_{a'} \to \psi_{a'} e^{-2i\alpha}$. Il n'est

donc pas possible si l'on conserve dans le Lagrangien $\mathscr{L}$ tous les vrais scalaires de généraliser la transformation (3.11) laissant $\mathscr{L}$ invariant ni par conséquent de définir U. Dans ces conditions la définition de la charge en fonction de I_3, U et N est, bien entendu, inacceptable. Mais il y a plus, car il n'est pas possible de trouver une transformation de jauge, c'est à dire de définir la charge. Ceci se voit par ailleurs d'une manière élémentaire sur l'expression (5.1): le premier terme conduit à des particules a' positives, le second à des particules a' (et non pas des antiparticules) négatives, ce qui est manifestement absurde surtout si l'on tient compte du fait que a' est de spin isobarique zéro.

Il existe deux attitudes possibles vis à vis d'un tel état de choses [16]). La première consiste à supprimer du Lagrangien tous les termes d'interaction qui empêchent une définition de U, donc de la charge. Un choix est alors nécessaire car il est permis, soit de supprimer le second terme de (5.1) et de considérer a' comme une particule positive, soit de supprimer le premier terme de (5.1) et de prendre alors a' négative. De toute façon de telles particules sont chargées bien qu'ayant $I = 0$. Il est évident que ce point de vue permet d'introduire dans la théorie un nombre considérable de types supplémentaires de particules.

La seconde attitude, plus radicale, consiste à supprimer d'office tous les champs tels que $\psi_{a'}$ qui conduisent à ce genre de difficultés. Ce point de vue introduit en fait un nouveau postulat, à savoir: "Un ensemble de champs n'est acceptable que quand tous les vrais scalaires, sans exception, formés à partir de ces champs peuvent, sans contradiction, figurer dans le Lagrangien".

Il ne semble guère possible à l'heure actuelle de prendre nettement parti en faveur de l'une ou de l'autre attitude. On peut simplement constater que le deuxième point de vue n'est pas en contradiction avec les faits présentement connus et que d'autre part il conduit à une présentation théorique plus élégante, et, sous certains aspects, plus prometteuse. En effet, il permet d'éliminer des particules chargées de spin isobarique zéro et surtout il apporte au nombre de types possibles de particules élémentaires des limitations qui peuvent s'avérer très utiles. Pour ces raisons il nous semble provisoirement plus satisfaisant.

Cependant il faut souligner que le problème du choix entre ces deux conceptions est d'ordre essentiellement expérimental.

Après ces discussions quelque peu détaillées concernant le champ

$\psi_{a'}$, mentionnons brièvement les conclusions se rapportant aux autres particules.

Des circonstances tout à fait analogues à celles rencontrées pour $\psi_{a'}$ se reproduisent si l'on introduit un champ $\psi_{b'}$ (spineur-isovecteur).

En ce qui concerne les mésons l'introduction d'un champ $\varphi_{\Delta'}$ (tenseur-isospineur de 2ème espèce) ne donne pas lieu à des difficultés. Ce champ décrit deux particules θ'^- et θ'° et leurs deux antiparticules $\overline{\theta'^-}$ (positive) et $\overline{\theta'^\circ}$. Il peut être montré que les θ' se comportent (du point de vue de la théorie ici discutée) exactement comme les $\bar{\theta}$. La présence de ces particules n'apporte donc rien d'essentiellement nouveau. Le champ φ_γ (tenseur-isoscalaire) peut également être introduit. Il conduit à des particules neutres se désintégrant très rapidement en γ ou mésons π. Par contre les champs $\varphi_{\delta'}$ (tenseur-isovecteur) et $\varphi_{\gamma'}$ (tenseur-isopseudoscalaire) donnent des difficultés très analogues à celles rencontrées pour $\psi_{b'}$ et $\psi_{a'}$. Si l'on adopte la seconde attitude discutée ci-dessus ces particules ne peuvent donc intervenir.

La discussion qui précède doit être complétée sur un point. Elle a été faite en prenant pour base les champs du tableau II et notamment ψ_a et ψ_b. Le second point de vue oblige alors à rejeter les champs $\psi_{a'}$ et $\psi_{b'}$. Toutefois il aurait été possible d'introduire initialement les champs $\psi_{a'}$ et $\psi_{b'}$ et ce même point de vue aurait alors conduit à supprimer ψ_a et ψ_b. Les conséquences physiques de la théorie n'en seraient d'ailleurs en rien modifiées. Autrement dit la présence simultanée des fermions isopseudoscalaire et isovectoriel est tout aussi permise que celle des fermions isoscalaire et isopseudovectoriel. C'est le mélange entre les deux couples qui conduit aux difficultés signalées.

En ce qui concerne les mésons, le fait que le méson π est une grandeur isopseudovectorielle conduit, toujours si la seconde attitude est adoptée, à ne pouvoir introduire qu'un champ isoscalaire (neutre) et à rejeter les champs isopseudoscalaire et isovectoriel; cette situation n'est pas sans analogie avec celle discutée au sujet des baryons.

Revenons maintenant à la première attitude qui consiste à supprimer certaines interactions au lieu de supprimer des champs. Pour montrer à quel point la théorie est alors malléable notons que si par exemple on ne conservait que le 2ème terme de (5.1) les particules a' seraient négatives avec "l'attribut" $a = N - U = 3$;

ce seraient donc les particules Y_E^- de Sachs [10]). Si au contraire seul le 1er terme de (5.1) était conservé les a' seraient positifs et correspondraient par leurs propriétés aux Λ^+ de Pais [6, 17]). Les autres particules de la théorie de Pais qui ne figurent pas dans le modèle de Gell-Mann seraient obtenues d'une manière analogue. Une différence importante par rapport à la théorie de Pais se présenterait cependant car les nouvelles particules seraient les duales (dans l'espace du spin isobarique) des anciennes et non simplement des états de charge supplémentaires. Il n'y a ici aucune raison pour que les masses de particules telles que Λ° et Λ'^+ soient égales.

Nous terminerons cette section par une remarque accessoire. Les valeurs de $(N - U)/2$ sont d'après (3.14) et (3.15) $0, \frac{1}{2}, \frac{1}{2}, 1, 0, \frac{1}{2}$ pour les particules N, $\Lambda, \Sigma, \Xi, \pi, \theta'$ respectivement†. Ces nombres sont égaux aux i^- correspondants de Pais. Pour obtenir à partir du formalisme de Pais tous les états de charge et les règles prédites par le modèle de Gell-Mann, et ceux là seulement, il suffit donc de modifier la définition (1.1) de la charge et de poser [16]) pour un système de particules

$$Q = I_3 - \Sigma i^- + \frac{N}{2};\ I_3 = \Sigma i_3^+,$$

i^+ étant le spin isobarique du modèle de Gell-Mann. Cette modification de la théorie de Pais comporte certaines difficultés qui ont été discutées ailleurs [16]). Superficiellement tout ceci n'est pas sans analogie avec une relation entre les théories de Pais et de Gell-Mann récemment signalée par Pais [17]) et par Utiyama et Tobocman [18]).

6. Conclusions

Grâce à l'introduction du postulat d'invariance par rapport aux retournements dans l'espace du spin isobarique (invariance du Lagrangien des interactions fortes) il a été possible de donner au modèle de Gell-Mann le support d'une théorie mathématique dont l'arbitraire est sûrement moindre que celui du modèle original. En effet, dans la définition de la charge (4.1), U n'est pas choisi pour chaque particule selon les besoins de la cause mais bien déduit du formalisme général qui conduit à la formule (3.15). Par ailleurs, en ce qui concerne la formule (4.1) elle-même on peut soit la regarder comme postulée à priori, et alors toutes les charges des baryons et mésons s'ensuivent, soit encore la considérer comme déduite de la

† Il a été dit plus haut que, physiquement, $\overline{\theta'}$ est parfaitement équivalent à θ et peut aussi bien être utilisé dans la théorie.

connaissance expérimentale des charges des nucléons et du Λ°. Enfin les interactions élémentaires découlent sans ambiguïté de la forme (3.1) du Lagrangien imposée par le postulat initial.

D'autre part il est naturel de s'attendre à ce qu'une formulation axiomatique apporte certaines restrictions au nombre des types de champs que l'on peut introduire librement dans la théorie. Au § 5 il a été cependant remarqué que le premier point de vue — qui consiste à supprimer certains termes d'interaction dus à la présence des champs supplémentaires et qui ne permettent pas de définir U — n'apporte en fait que de très faibles restrictions. A titre d'exemple citons le cas de la particule a' qui, suivant les interactions que l'on conserve, est soit positive (analogue au Λ^+ de Pais) soit négative (le Y_E^- de Sachs). Dans le modèle de Gell-Mann il est parfaitement concevable d'introduire simultanément ces deux particules différentes avec $Q = I_3 + 1$ et $Q = I_3 - 1$ respectivement. Au contraire, d'après la première attitude du présent formalisme il est nécessaire de faire un choix sans quoi une particule ne serait pas complètement définie par le caractère de transformation de son champ (voir § 3*a*). Néanmoins une de ces particules peut être introduite dans la théorie.

Une restriction beaucoup plus sévère est apportée par la deuxième attitude discutée au § 5, et selon laquelle un ensemble de champs n'est acceptable que quand tous les vrais scalaires, sans exception, formés à partir de ces champs peuvent, sans contradiction, figurer dans le Lagrangien. Alors un champ tel que $\psi_{a'}$ doit être necessairement éliminé. Plus généralement on constate que pour tous les quanta des champs acceptables $|U| \leqq 1$. Ceci d'ailleurs est satisfaisant car si l'on se rapporte à la formule (3.15) et à l'interprétation (4.3), $|U| \leqq 1$ élimine les "double-isofermions" etc.

Exprimant cette condition en terme d'étrangeté ou d'attribut, on obtient

$$\begin{aligned} |a| = |s| = |U - N| &\leqq 1 \qquad \text{pour les bosons,} \\ &\leqq 2 \qquad \text{pour les fermions.} \end{aligned}$$

Si donc ce deuxième point de vue est adopté il faut conclure que l'on a déjà observé des particules ayant l'étrangeté maximum. Toutefois, comme seules les propriétés des champs dans l'espace du spin isobarique ont été discutées dans cet article, ceci ne doit pas signifier que toutes les particules appartenant à l'un des types de champs des tableaux II ou III soient de même nature. Etant

donné que la théorie dispose des nombres quantiques issus de l'invariance dans l'espace ordinaire une differentiation supplémentaire est naturellement possible [18, 19]). Il faut cependant noter que dans ce deuxième point de vue le nombre des champs possibles sera, malgrè tout, assez restreint. Il y a peut être là un principe de limitation du nombre des types de particules élémentaires susceptible d'applications.

Pour cette raison, et pour les motifs mentionnés ci-dessus, la seconde attitude, qui ne conduit à aucun désaccord avec les données expérimentales actuelles, nous semble provisoirement préférable.

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OBSERVATIONS OF THE ENERGY-SPECTRUM OF THE COSMIC RADIATION BELOW GROUND

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Abstract: In nuclear research emulsions exposed at a depth of 57 mwe the momentum of 400 particles was determined by multiple scattering measurements on long tracks. 317 occurred within 30° of the vertical, and for this group the momentum spectrum in the range $0 < p < 40$ GeV/c was found to be consistent with a law of the form usually assumed $N(p) \sim (p + p_0)^{-\gamma}$ with $p_0 = 12$ GeV/c and $\gamma = 3$. Particular attention was paid to the momentum interval 0 to 1 GeV/c and no evidence of any anomalies found.

The distribution in angle of the tracks was found to be consistent with a law of the form $\cos^n \theta$ with $n = 2.7 \pm 0.2$. For those tracks with grain density at least 1.2 times the plateau value (slow tracks) the exponent was found to be 3.7 ± 0.25. These values correspond to exponents of the cosmic radiation equal to 1.7 ± 0.2 for the fast tracks and 2.7 ± 0.25 for the slow.

1. Introduction

Observations of the momentum distribution are fundamental to our understanding of the cosmic radiation. Observations at sea level were first made by Blackett [1]) and have been repeated by many workers, the most recent and extensive results having been obtained by Owen and Wilson [2]).

Measurements at places other than at sea-level have been scanty and of much lower accuracy. In this connection the measurements of Adams *et al.* [3]) at airplane altitude and of Miller *et al.* [4]) at mountain altitude may be noted.

In particular, no measurements have been made below ground. On the other hand, such measurements are needed in order to form a basis for the interpretation of underground observations of large-angle scattering by George, Redding and Trent [5]) and Kannangara and Shrikantia [6]) and of nuclear interactions by Cocconi and

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Cocconi [7]), George and Evans [8, 9]) and of Annis, Wilkins and Miller[10]). For example, George, Redding and Trent had shown that the observed large-angle scattering of the underground radiation was in excess of that calculated. The calculated figure was obtained by averaging the expected Coulomb scattering over the energy spectrum of the radiation. In the absence of any direct measurements, the spectrum had to be assumed. The usual assumption was made that the spectrum below ground was simply that at sea-level modified purely by ionisation losses in the overlying earth. One possible interpretation of the anomalously high frequency of large angle scatters observed below ground was that the assumed energy spectrum was incorrect, there being a much larger number of low energy particles than that assumed, the ordinary Coulomb scattering of these extra-particles being sufficient to account for the observations.

In order to test this hypothesis in particular, and in order to determine the energy spectrum over as wide an energy range as possible, multiple scattering measurements were made on long tracks in emulsions exposed below ground. The results are presented in this paper.

2. Experimental

Ilford nuclear research emulsions, type G5 and 200 microns thick were used. In all, three separate batches were employed and the relevant details are:

Batch No. 1. Poured, stored and processed at room temperature underground. Fast and slow tracks recorded.

Batch No. 2. Poured at Ilford and transferred immediately in horizontal position to underground laboratory. Stored vertically underground. Processed at sea-level by temperature cycle method. Fast tracks only recorded for momentum spectrum measurements.

Batch No. 3. Poured, stored and processed at 4° C underground. Stored close under tunnel roof. Fast and slow tracks were recorded.

By "fast" tracks we mean those in which the grain density is not more than 20 % above the plateau value. The great majority of tracks in these underground emulsions fell into this category. Occasionally one sees a track significantly above this value and these we call "slow" tracks. Very few tracks were observed near the border-line.

The underground exposures were made in a disused tunnel of the underground railway in London, at Holborn station, at a depth equivalent to 57 m water. During the underground exposures, these plates were stored with the emulsion plane vertical in sealed containers for a period of four months. The plates of batch No. 2 were kept horizontal throughout their history above ground, which was less than 24 hours. In view of the $\cos^2\theta$ angular distribution of the sea level radiation, it follows that the contribution of long fast tracks recorded at sea level is negligible.

The plates were cut into squares 5 cm × 5 cm and for the fast track analysis one sweep 3 cm long in the middle of each plate was made using a × 45 oil immersion objective, leaving off 1 cm from each edge. Only straight through tracks of length greater than 1 cm were recorded. On all such tracks, the angle with respect to the vertical, the grain density and the mean angle of multiple scatter using Fowler's [11]) method were determined.

In plates from batches 1 and 3, which were poured, stored and processed underground, a search was also made for slow tracks of grain density greater than 1.2 times the plateau value, and of length greater than 4.5 mm. These were originally investigated in order to determine the masses of the particles in the underground radiation but they also give useful information on the low-energy end of the momentum spectrum. This latter aspect of the investigation of the slow particles is included in this paper.

For the slow particle analysis, scanning was done along lines 5 mm apart, leaving off 5 mm at each edge of the plate.

3. Results

3.1. ANGULAR DISTRIBUTION OF LAST TRACKS

A total of 287 fast tracks was analysed. A histogram giving the distribution in the angle with respect to the vertical is given for these tracks in fig. 1(a). Assuming that the angular distribution obeys a law of the form $I(\theta) = I\,(0)\cos^n\theta$, the best value of n fitting the observations was determined by least squares. The value so found was

$$n = 2.7 \pm 0.2. \qquad (1)$$

3.2. ANGULAR DISTRIBUTION OF SLOW TRACKS

The total number of slow tracks was 106. Their angular distribution is shown in fig. 1(b). Assuming the same form of angular

distribution, the best fit to the results is obtained for the value

$$n = 3.7 \pm 0.25. \qquad (2)$$

Fig. 1(a)

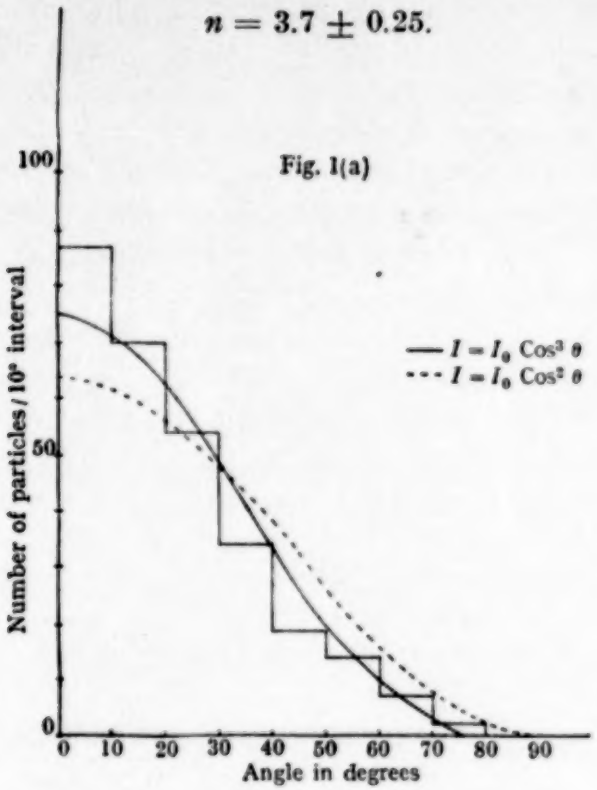
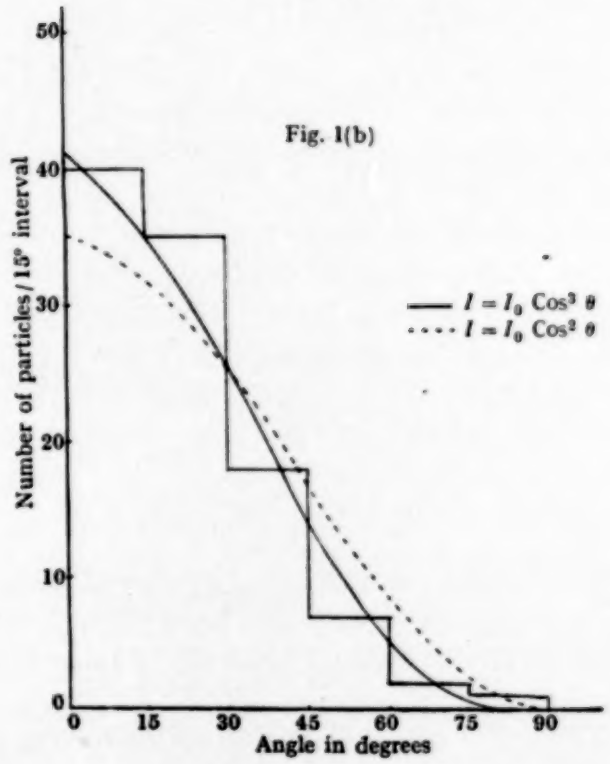


Fig. 1(b)



3.3. MOMENTUM DISTRIBUTION OF FAST TRACKS

The multiple scattering was measured in the usual way by taking eyepiece scale-readings at regular intervals along the track. The cell-length over which readings were taken was usually 1 millimetre, and at the end of each millimetre the mean of 10 closely spaced readings was taken in order to reduce the reading errors. The readings were made using a Cooke, Troughton and Sims M4000 microscope. The noise level of the readings, including the reading errors and effects of stage-nonlinearity was found to be 0.2 microns. In determining the multiple scattering-angle, α, a cell length, S, was chosen so that the mean value of second difference of the coordinate readings was greater than 1 micron, thus ensuring that the effects of noise were negligible. The value of the $p\beta$ of the tracks was found in the usual way from

$$p\beta = \frac{KS^{\frac{1}{2}}}{\bar{\alpha}}. \tag{3}$$

The value of K, the scattering constant, used was that given by Voyvodic and Pickup [12]):

$$K = 2\mathbf{r}^2(\Sigma N_i Z_i^2)^{\frac{1}{2}}\left\{1.45 + 0.8\left(\ln\frac{0.94S}{\beta^2 + 0.3}\right)^{\frac{1}{2}}\right\}, \tag{4}$$

where N_i is the number cm^{-3} of emulsion nuclei of atomic number Z_i. For the fast tracks, β was sufficiently close to unity, and only the slow dependence of K on cell length was taken into account. It is also obvious that for these tracks, the quantity $p\beta$ is a measure of the momentum p.

The distribution in momentum, determined in this way, of particles whose inclination to the vertical is less than 30° is shown in fig. 2(a).

In the past, it has been customary to follow Euler and Heisenberg [13]) and to assume that the differential momentum spectrum in the vertical direction is of the form

$$N(p)dp \sim \frac{dp}{(p + p_0)^{\gamma}}, \tag{5}$$

where p_0 is the total momentum loss by ionisation travelling vertically to the point ot observation, and γ is a constant approximately equal to 3. In our case p_0 is not known exactly due to uncertainties in the correct value of energy loss at high energies, but is known to lie in the region 12 to 15 GeV/c. Accordingly, we tried to fit a

law of the form of eq. (5) to our results. Unfortunately, it turns out that with two parameters p_0 and γ available, the results may be equally well fitted by a whole range of pairs of values, from $p_0 = 12$ GeV/c, $\gamma = 3.1$ to $p_0 = 24$ GeV/c, $\gamma = 5.7$. Several intermediate values were also tried, the relation between corresponding

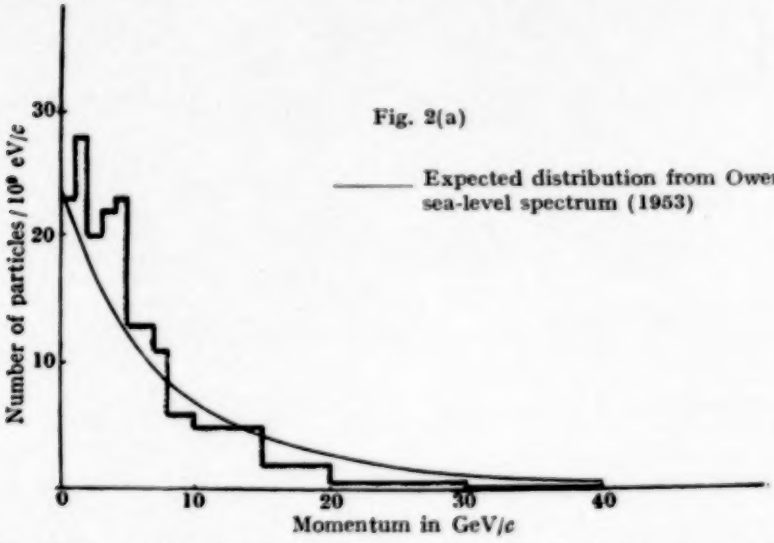


Fig. 2(a)

——— Expected distribution from Owen's sea-level spectrum (1953)

values of p_0 and γ that fit the observations being $p_0 = 4\gamma$, with p_0 in GeV/c. On the side of high values of p_0, there is no limit. On the low side, the log-log plot begins to show marked signs of curvature for $p_0 = 6$ GeV/c. All this means that the present measurements, while being consistent with the usually accepted law and values of p_0 and γ, are not any good for establishing independently the values of these constants.

Also drawn in fig 2(a) is a curve showing the distribution expected from the measurements of Owen and Wilson [2]). This will be discussed later.

3.4. THE MOMENTUM DISTRIBUTION AT LOW VALUES OF MOMENTA

A histogram showing the frequency distribution of the 106 slow tracks as a function of $p\beta$ is shown in fig. 2b. The distribution is fairly flat in the middle with a sharp cut-off at each end. These cut-offs are due simply to the selection criteria, the cut-off at low momenta being due to the fact that only tracks > 4.5 mm were accepted, and at high momenta to the requirement of grain density greater than 1.2 times plateau value. These tracks were found in a total scan of 3584 cm.

In order to obtain the spectrum in the range 0—1GeV in greater detail, the results for the slow tracks were joined on to the first interval (0—1GeV/c) of the fast track distribution as follows.

We first of all express all particle fluxes as so many tracks per unit momentum interval, per cm of scan. In doing this for the slow

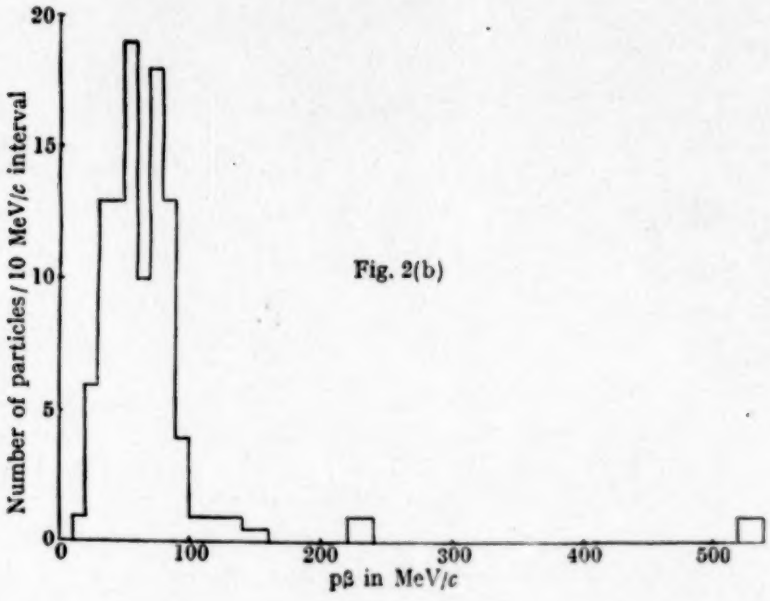


Fig. 2(b)

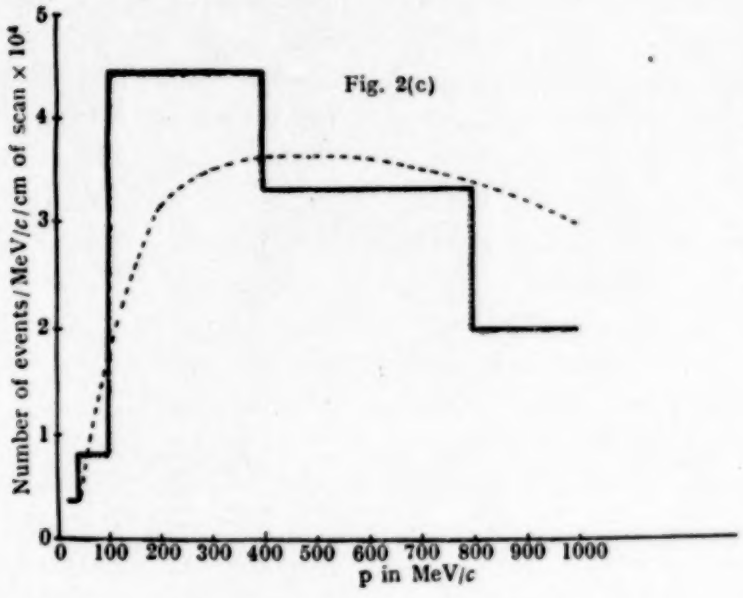
tracks we must allow for the fact that 25 % of the grey tracks crossed two scanning lines and 5 % crossed three scanning lines, making the effective length of scan 3584/1.6 = 2240 cm. Also, we must only consider those tracks whose length in the emulsion exceeded 10 mm and which were within 30° of the vertical. There were 11 such particles. Therefore, from fig. 2(a) we conclude that in the momentum range 40—80 MeV/c, there were

$\frac{11}{106} \times \frac{1.3}{2240} = 6.10^{-3}$ particles/MeV/c/cm of scan leaving tracks longer than one cm in the emulsion. In the first group of the fast tracks, there were 10 in the momentum range 100—400 MeV/c, 10 in the range 400—800 MeV/c, and 3 in the range 800—1000 MeV/c. These were found in a scan of 75 cm. These have also been expressed as the number per MeV/c per cm of scan, and the result for the two sets of tracks is shown in fig. 2c.

The expected distribution, arrived at in a manner discussed below is drawn in for comparison. The observed results are not inconsistent with the calculated values.

4. Discussion

The two results obtained concerning the cosmic radiation at a depth of 57 mwe are the angular distributions: $n = 2.7 \pm 0.2$ for fast tracks and 3.7 ± 0.25 for slow tracks, and the momentum spectrum shown in fig. 2(a) and 2(c). These will now be discussed separately.



4.1. ANGULAR DISTRIBUTION

(a) *Fast tracks*

It is well known that over a wide angular range, the angular distribution of the cosmic radiation can be fitted to a simple power-law in $\cos\theta$. Let β be the true exponent, i.e. the exponent in the distribution of the flux at direction θ per unit area at right angles to the direction θ. It will be readily understood that the present method, which measures the flux of particles which cross a fixed horizontal plane in the emulsion, will give an angular distribution in the exponent of $\cos\theta$ equal to $\beta + 1$, i.e. our measured valves have to be reduced by unity to give the experimental value for the true exponent β. The true exponent is measured by rotating a telescope of short cylindrical counters around a horizontal axis. Using this method, Follett and Crawshaw [14]) obtained the value $\beta = 2$ at the same depth below ground, and our value for β for the fast tracks, 1.7 ± 0.2 is not inconsistent with this.

Goldsack and Kannangara [15]) obtained the value $\beta = 5$ using

emulsions exposed in the same underground laboratory. They concluded that their emulsion searching was biased in favour of tracks close to the vertical. As our value of β is close to the expected value, we conclude that any such bias in our track selection is small.

(b) *Slow tracks*

The measured exponent for the slow tracks is 3.7 ± 0.25 corresponding to a value of β for the slow tracks of 2.7 ± 0.25. This is significantly greater than the value for the fast tracks, the difference between the values of β for slow and fast tracks being unity.

This difference arises from the fact that the momenta of the slow tracks lie within a small differential interval, while for the fast tracks, the momenta can have any value greater than a certain minimum value, which, in our case was 120 MeV/c for μ-mesons. A simple argument, deriving the angular distribution from the momentum distribution will illustrate this point. We assume that the primary particles giving rise to the μ-mesons of momentum greater than 12 GeV/c recorded in our laboratory, are isotropic in space. Then the distribution in momentum, at the angle θ, corresponding to equation 5 which refers to the vertical direction will be

$$N(p, \theta)\, dp \sim \frac{dp}{(p + p_0/\cos\theta)^{\gamma}}. \tag{6}$$

For values of $p \ll p_0$, this becomes

$$N(p, \theta)dp \sim \cos^{\gamma}\theta\, dp, \tag{7}$$

whereas, integrating (6) from a minimum value p_1 to infinity, we see that for $p_1 \ll p_0$,

$$N(> p_1\ \theta) \sim \cos^{\gamma-1}\theta. \tag{8}$$

For the slow tracks analysed here, $p \approx 100$ MeV/c, $p_0 \approx 12$ GeV/c and eq. (7) applies. Similarly eq. (8) will correspond to our fast tracks since $p_1 = 120$ MeV/c. The difference between the observed exponents, 2.7 ± 0.25 for the slow tracks and 1.7 ± 0.2 for the fast tracks can then be understood in the light of eq. (7) and (8).

Furthermore, the exponents of the angular and momentum distributions of the fast tracks should be the same.

Here we have ignored the effects of the nuclear absorption of

π-mesons or the angular distribution below ground. Barrett *et al.* [16]) have shown that this effect is always small, and that at our depth of observation it is negligible.

4.2. MOMENTUM DISTRIBUTION

The momentum distribution of the cosmic radiation has been accurately measured only at sea level. The results presented here are the first measurements of the distribution below ground and are of somewhat limited statistical accuracy. A total of 317 tracks has been used in the analysis and the work of finding and measuring up these tracks occupied more than two years. No very great improvement in the statistical accuracy can be expected using emulsion techniques. Large numbers of events can only be anticipated from magnetic spectrographs such as that of Caro, Parry and Rathgeber [17]) and of Owen and Wilson [2]). It should be noted, however, that, except at extremely low momenta, no bias is introduced in selecting tracks in the emulsion method. This is not so in the spectrograph method, where single particles, unaccompanied by other charged particles within the sensitive area of the apparatus are selected for measurement. Since the probability of not being accompanied is undoubtedly energy-dependent, some bias in the selection of particles must be introduced.

In the absence of any underground energy-spectrum measurements, we have compared the results with a curve of the form (5) suggested by Euler and Heisenberg, but the constants p_0 and γ are not uniquely determined.

Another line of approach is to compare our results with the sea level data of Owen and Wilson [2]). Their differential spectrum was integrated to give the integral spectrum. We then read off from this curve the momentum corresponding to the directly observed relative intensity below ground of 6.9 %. This latter figure was obtained by taking the counting rate above and below ground of a coincidence telescope containing 10 cm of lead. This value of momentum was found to be 13.5 GeV/c. Assuming that all the particles below ground have arrived from above ground, and that they all have suffered the same momentum loss by ionisation then the expected spectrum would simply be the Owen and Wilson figures with the momentum origin shifted to the value 13.5 GeV/c. The curve so obtained is plotted in fig. 2(a) for comparison with our observations. The calculated curve has been normalised to 211

particles. The general trend of the two results are seen to be very similar. There is some indication of an excess of particles observed for $1 < p < 5$ GeV/c, and a deficit for $p > 20$ GeV. In view of the limited statistical accuracy of the observations it is not possible to regard these discrepancies as established.

The low-momentum end of the momentum distribution must be considered separately, on account of variations in the momentum interval occupied by the same group of particles above and below ground, due to non-linearity of the range momentum relation.

The calculated differential spectrum underground allowing for this effect is

$$N(p) \sim \frac{1}{(p + p_0)^{\gamma}} \frac{\left[\frac{dR}{dp}\right]}{\left[\frac{dR}{dp}\right]_{p=p+p_0}}. \tag{9}$$

For dR/dp we have taken the form given by Rossi and Greisen [18])

$$dR/dp \sim \frac{(p/\mu c)^3}{[1 + (p/\mu c)^2]^{\frac{3}{2}}}, \tag{10}$$

where μ is the meson mass. For our conditions of observation, we have taken $p_0 = 13.5$ GeV/c, $\gamma = 3$. The (dR/dp) in the denominator in eq. (9) is a constant, and the (dR/dp) term in the numerator only makes itself felt for $p < 3\,\mu c$. The spectrum calculated using eq. (9) and (10) has been plotted in fig. 2(c) for comparison with the observations. The agreement between the observed and calculated values is seen to be satisfactory.

This general agreement between the observed and expected momentum distributions, particularly in the 0 to 1 GeV/c range, means that we can rule out the possibility of explaining the observed excess of large-angle scattering in lead of the underground radiation in terms of an excess of low-energy particles which suffer normal coulomb scatter. This possibility was admitted by George, Redding and Trent [5]) and again emphasized by Cooper and Rainwater [19]). There is general agreement that the μ-mesons of the penetrating component are scattered by complex nuclei as they would be if the charges on these nuclei were concentrated in a point. On the other hand, observations of the scattering of electrons, of μ-mesic x-rays, and of the coulomb energy of nuclei all point to a distribution of charge which should give a much lower frequency of large-angle scatter. The only possibilities left now seem to be that

either the so-called anomalous scattering of μ-mesons is in fact Coulomb scattering, the existing theories being inadequate, or that there is some additional non-Coulombian coupling between μ-mesons and nucleons.

5. Conclusions

From the observations reported in this paper, the following conclusions have been drawn concerning the cosmic radiation at a depth of 57 mwe:

(i) In the momentum range $1 < p < 40$ GeV/c, the differential momentum distribution is not inconsistent with a law of the usual form $N(p) \sim (p + p_0)^{-\gamma}$, with $p_0 = 12$ GeV/c and $\gamma = 3$.

(ii) The present observations only determine a range of values for the constants p_0 and γ. Values which give a satisfactory fit varied from 12 to 24 GeV/c for p_0, with the corresponding value of γ being approximately equal to $p_0/4$.

(iii) In the momentum range $0 > p > 1$ GeV/c, the observed distribution is not inconsistent wtih the expected distribution. The low intensity of the slow particles is accounted for by variations in the momentum interval occupied by the same group of particles above and below ground.

(iv) Any explanation of the observed frequency of large-angle scatters of cosmic-ray particles underground in terms of a significant departure of the momentum distribution from that usually assumed can now be ruled out. The observed distribution is in general agreement with that usually assumed.

We are grateful to Professor Messel for providing excellent facilities for the work. We are indebted to Professor Wilson and Dr. Owen for sending us their results prior to publication. One of us (G.S.S.) also wishes to acknowledge the grant of Research Studentships from the J. N. Tata Endowment, Bombay, and the Research Committee of the University of Sydney during different stages of the work. We would like to thank the London Transport Executive for permission to expose the plates at Holborn Station.

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PROGRESS REPORT

NEW RESULTS ON ELEMENTARY PARTICLES AT THE PISA CONFERENCE (12—18 JUNE, 1955)

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A major part of the experimental section of the Pisa Conference was concerned with the possible mass differences of the K mesons which decay in various modes. The two main sources of experimental material were provided by the eagerly awaited results of the machine experiments at Berkeley and the cosmic ray work using very large emulsion stacks.

The Berkeley experiments were performed with emulsions only and the analysis carried out at various places. Particles of a narrow band of momentum were allowed to fall on stacks of stripped emulsions. Since the range of K mesons of a given momentum is substantially different from those of protons or π-mesons of the same momentum, the K mesons are readily detected. Measurements of momentum versus range give one in principle an accurate value of the mass, but the range-energy relation for emulsions is at present not sufficiently well known for these direct mass values to be of great significance. However, the difference of mass between K particles decaying in different modes can be determined without an accurate knowledge of the range-energy relation. The very preliminary results for these mass differences reported at Pisa are given in Table I.

These do not indicate any difference of mass. The K_μ is the most common mode of decay of K-mesons, hence the evidence presented in Table I indicates that the masses of the K_μ and τ particles are nearly equal. The mass of the τ is known certainly to within $2m_e$ (from its characteristic decay) and its mass is not very sensitive to the range-energy relation for emulsion. The mass estimate of the K_μ obtained from multiplate chamber work on S-particles is somewhat lower but these determinations depend critically upon the range-energy relations in the various absorbers, mainly copper.

TABLE I

Decay mode	Mass difference $m_K - m_\tau$
(*a*) Berkeley Results	
τ'	-2.4 ± 9.8
$\varkappa$	12.0 ± 16.0
K_μ	-1.8 ± 10.8
χ	-9.0 ± 13.0
K_{L3}	-38.0 ± 21.0
(*b*) M.I.T. group	
K-mesons not separated in different types	-5 ± 5

Preliminary measurements on the interaction and decay in flight in the emulsions of positive mesons produced at the Berkeley machine have also been presented giving the following values:

$$\lambda_{K^+} = 100 \text{ cm emulsion}$$

$$\tau_{K^+} = 1.1 \pm 0.3 \times 10^{-8} \text{ sec.}$$

Out of 18 inelastic events, the K^+ re-emerges in five cases and in no other case did the energy release exceed the kinetic energy of the primary particle.

TABLE II

Ranges of Stopped Secondaries

	M.I.T.	E.P. Pic du Midi	E.P. Chamonix	G-stack emulsion	E.P. emulsion	Berkeley emulsion
K_μ	75.9 ± 1.8 g/cm² Cu	75.8 ± 1.9 g/cm² Cu	102.2 ± 2.5 g/cm² Pb	20.52 ± 0.2 cm	20.33 ± 0.33 cm	20.4 ± 0.4 cm
			77.1 ± 3 g/cm² Cu	78.2 ± 1.4 g/cm² emulsion	78.3 ± 1.9 g/cm² emulsion	78.5 ± 0.2 g/cm² emulsion
χ	44.2 ± 1.4 g/cm² Cu		59.8 ± 1.6 g/cm² Pb	11.78 ± 0.02 cm	11.71 ± 0.038 cm	
			44.7 ± 1.7 g/cm² Cu	44.9 ± 0.8 g/cm² emulsion	44.6 ± 1.3 g/cm² emulsion	

Other important contributions included the analysis of the secondaries of K-particle decays. The main results concerned the ranges of stopping secondaries in emulsion and come chiefly from the G-stack collaboration and from the Ecole Polytechnique Group. Table II contains the ranges of stopping secondaries presented at the Pisa Conference. This table should be compared with the values obtained by the cloud-chamber for S-particles.

These results finally establish the two body decay schemes

$$\chi \rightarrow \pi + \pi^{\circ},$$

$$\varkappa_{\mu} \rightarrow \mu + \nu.$$

The association of π° mesons as opposed to γ-rays with S-particles that are not $K_{\mu 2}$ has been already shown by the work of the M.I.T. group. Finally an example from the G-stack of the decay of a χ meson decaying to $\pi^{+} + e^{+} + e^{-} + \gamma$ showed the decay scheme that had been assumed for some time to be in fact correct.

One might remark on the consistency of the ranges of the various secondaries obtained in photographic emulsions. Unfortunately our lack of knowledge of the range-energy relations prevents us from getting the desired accuracy of the masses of the decaying $K_{\mu 2}$ from these ranges of their secondaries. There is every reason to hope that the range-energy relation for emulsions will be determined properly in the near future.

Little additional information was added to our knowledge of $\varkappa$ and K_{β}. It is obvious that the form of the energy spectrum of their charged decay products will soon be known.

Concerning the relative frequencies of the different modes of decay of K-particles the effect of scanning bias was discussed. Table III summarizes the information presented at the conference on the relative frequency of different modes of decay as observed in emul-

TABLE III

Relative Frequencies of K-meson Decay Modes

(G-stack experiment)

$\frac{K_{\beta}}{K_{\mu}}$	$\frac{\varkappa}{\chi}$	$\frac{K_{\beta}+\varkappa}{K_{\mu}+\chi}$	$\frac{K_{\beta}+\varkappa}{\text{all K's}}$	$\frac{K_{\mu}}{\chi}$	$\frac{K_{\beta}}{\varkappa}$
0.17	0.47	0.26	0.21	2.1	0.8

sions. Here too an enormous advance is to be expected with the available K beams but it must be remembered that the moderation

time for K-mesons for machine exposed emulsions and cloud chambers is of the order of 10^{-8} sec, while for cosmic ray stacks and the stacks exposed to proton and π beams it is only of the order of 10^{-10} sec.

The evidence presented on K^- remains more meagre than that for positive K's. At the Pisa Conference the majority of information on K^- interactions came from cosmic ray exposed stacks. 22 K^- captures were reported from stacks exposed to a K^-beam. In these plates decays and interactions in flight were observed giving the first clear evidence as to the lifetime of the negative K^-'s, the most probable value being 0.6×10^{-8} sec, based unfortunately on only three decays. The study of the interaction in flight of negative K mesons shows that the visible energy release exceeds the kinetic energy of the incoming particle in approximately half the cases. This is to be compared with the predictions of Gell-Mann and Nishijima. Further examples on the emissions of hyperons and on hyperfragments from K^- capture stars, were reported. Perhaps we should remark on the beautiful example reported by Teucher in which a Λ° emitted from the capture star decayed after a distance of 27μ from the origin.

τ-mesons

Our source of information on τ-mesons still came almost exclusively from cosmic radiation, since the τ-meson examples found in machine plates had at the time of the Pisa Conference not yet been fully analysed. Amaldi presented the combined results of τ-mesons observed in cosmic radiation, and discussed the various possible assignments of spin and parity based on the theoretical considerations of Dalitz and Fabri. Any conclusion that one can reach depends strongly upon the low energy π from the decay. It is clear that these examples may not be free from experimental bias. The simplest assumption is 0^- for the τ-mesons and the experimental evidence does not contradict it.

Hyperfragments

One session was devoted to hyperfragments and a number of interesting individual examples were reported. Outstanding were no fewer than three examples of the decay of

$$^*H^4 \rightarrow \pi^- + He^4 + 55 \text{ MeV}.$$

Table IV gives the relevant details and illustrates the beautiful agreement among the three cases. Equally interesting were two examples of $^*He^5$: $^*He^5 \rightarrow He^4 + p + \pi^- + 34$ MeV. These were the first examples of $^*H^4$ and He^5 to be reported.

TABLE IV

$^*H^4_1$ — Hyperfragments

Event	Range of α	Range of π
Bristol	8.5μ	38.22 mm
Ecole Polytechnique	8.7μ	>34.30 mm
Oslo	8.3μ	37.6 mm

Associated production

Ample confirmation for pair production of strange particles, predicted by Nambu *et al.* and Pais, in order to account for copious production and long lifetime, continues to come forth. The evidence accumulated in cloud chambers, emulsions and machines shows that all types of K^+, with the exception of K_β, have been observed definitely to be associated either with hyperons or with hyperfragments. More startling is the fact the only other strange particle to be produced with a K^- has been a second K. A very beautiful example has emerged after re-examination of cloud chamber photographs from California Institute of Technology which shows the simultaneous production of $\Xi + \theta^\circ + \bar{\theta}^\circ$.

BOOK REVIEW

RAYMOND L. MURRAY., *Introduction to nuclear engineering* (George Allen and Unwin, London. 1955. ix-418 pages. 30s.)

Although the subject of this book falls outside the scope of the journal, I should like to call its existence to the attention of the readers, since it may be very handy to those of them who may want a quick introduction to the subject of Nuclear Engineering. The work under review is an elementary textbook of the usual matter-of-fact type characteristic of American production. It adequately covers in twenty chapters the various scientific and technical problems involved in the construction and operation of the different types of reactors. It only gives the bare outline of the subject, but it does this in a very clear and neat fashion. Each chapter is accompanied by a selection of problems and adequate references to articles and books for further reading. It is primarily destined to engineering students, but it will also have its usefulness in a limited way for research workers in Nuclear Physics confronted with some problem pertaining to the engineering aspect of nuclear energy production.

This is the English printing of a book originally published in the United States. It is well produced and has excellent diagrams and other illustrations.

L. R.

NOTE ON NOMENCLATURE

Since the nomenclature in the field of Nuclear Physics is not yet quite unified and a number of illogical expressions are unfortunately still widely used, it is advisable to give the following indications about the terminology adopted in this journal.

(1) The word 'electron' applies to particles with an elementary charge of either sign. When it is desirable to emphasize the sign of the charge, the words 'positon' and 'negaton' are used. Thus the illogical phrase 'electron-positron pair' is replaced by 'electron pair', or, if required, by 'positon-negaton pair'.

(2) Nuclei (or nucleids, *not* 'nuclids') are conveniently divided according to the parity of the proton and neutron numbers Z, N, into four families as follows:

even mass nuclei	{ even nuclei (Z even, N even) odd nuclei (Z odd, N odd)
odd mass nuclei	{ odd proton nuclei (Z odd, N even) odd neutron nuclei (Z even, N odd).

(3) The expressions 'isotopic spin', 'isotopic quantum number' are replaced by 'isobaric spin', 'isobaric quantum number'.

(4) For the elementary particles use is made of the latest nomenclature adopted by international agreement.

BETA- AND GAMMA-RAY SPECTROSCOPY

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A STATEMENT BY W. H. ZINN

Director, Argonne National Laboratory, Lemont, Illinois

The techniques and products of atomic energy are receiving ever wider application in scientific research, in medical treatment, and in the control of industrial processes. The use of radioactive materials is becoming almost commonplace. The large-scale generation of electricity with nuclear reactors is being developed in many countries through vigourous programmes of research and construction. It is probable that the years just ahead will find ever more numerous organizations and individuals newly engaged in the study and use of atomic energy.

These circumstances indicate that it is timely to initiate a series of authoritative texts dealing with the applications of nuclear science in research and in industry. Through its various series in science and engineering, the McGraw-Hill Publishing Company has played a vital part in the education of scientists and engineers and in supplying them continuously with scholarly appraisals of the latest results in rapidly growing fields of research. It is my expectation that this new Series in Nuclear Engineering will be in the tradition of the previous McGraw-Hill series and that it will contribute significantly to the development of this newest technology.

Documents prepared by the U.S. Atomic Energy Commission

TO BE PUBLISHED THIS AUTUMN

This collection of selected reference materials was assembled by the United States Atomic Energy Commission to help scientists and technicians from other nations interested in the development and use of nuclear energy. Sets of these documents were sent to the delegates of each country attending the International Conference on the Peaceful Uses of Atomic Energy held at Geneva in August this year. This material provides a practical basic source of technical information on nuclear energy and its applications. It is now being published by McGraw-Hill in six volumes:

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